Introduction to Abstract Categorial Grammars

First Course

Motivations

The Categorial Grammar tradition

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- a type-theoretic view of grammars and grammatical composition

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  - a (small) finite set of atomic types
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- grammatical composition as a typing discipline
  - grammatical composition obey the rules of a given typing system
  - the rules of grammatical composition are completely specified by the underlying type calculus

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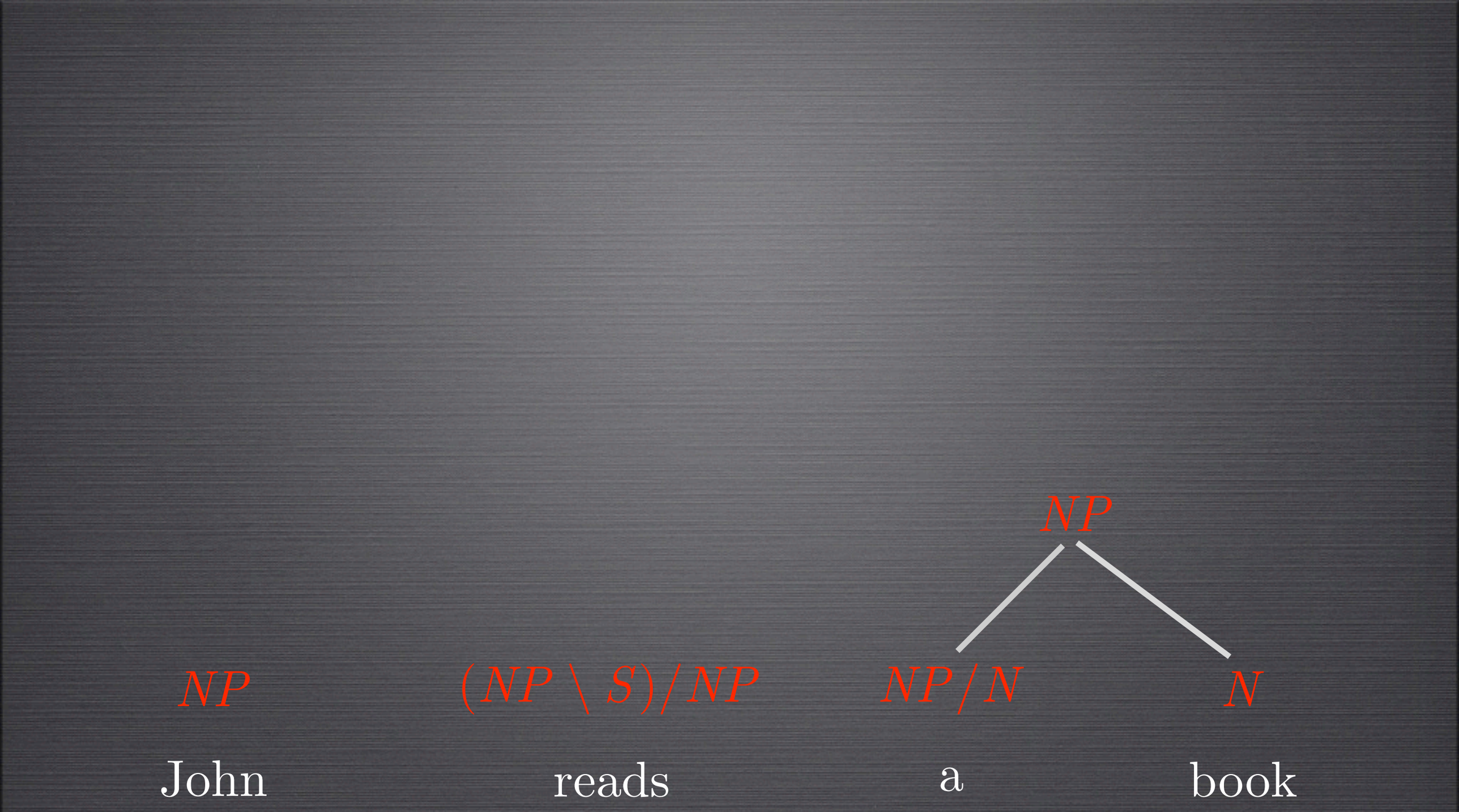
John	:	NP
reads	:	$(NP \setminus S)/NP$
a	:	NP/N
book	:	N

Type calculus: $\alpha,\ \alpha\backslash\beta \vdash \beta$

$\beta/\alpha,\ \alpha \vdash \beta$

John reads a book

NP	$(NP \setminus S)/NP$	NP/N	N
John	reads	a	book


NP
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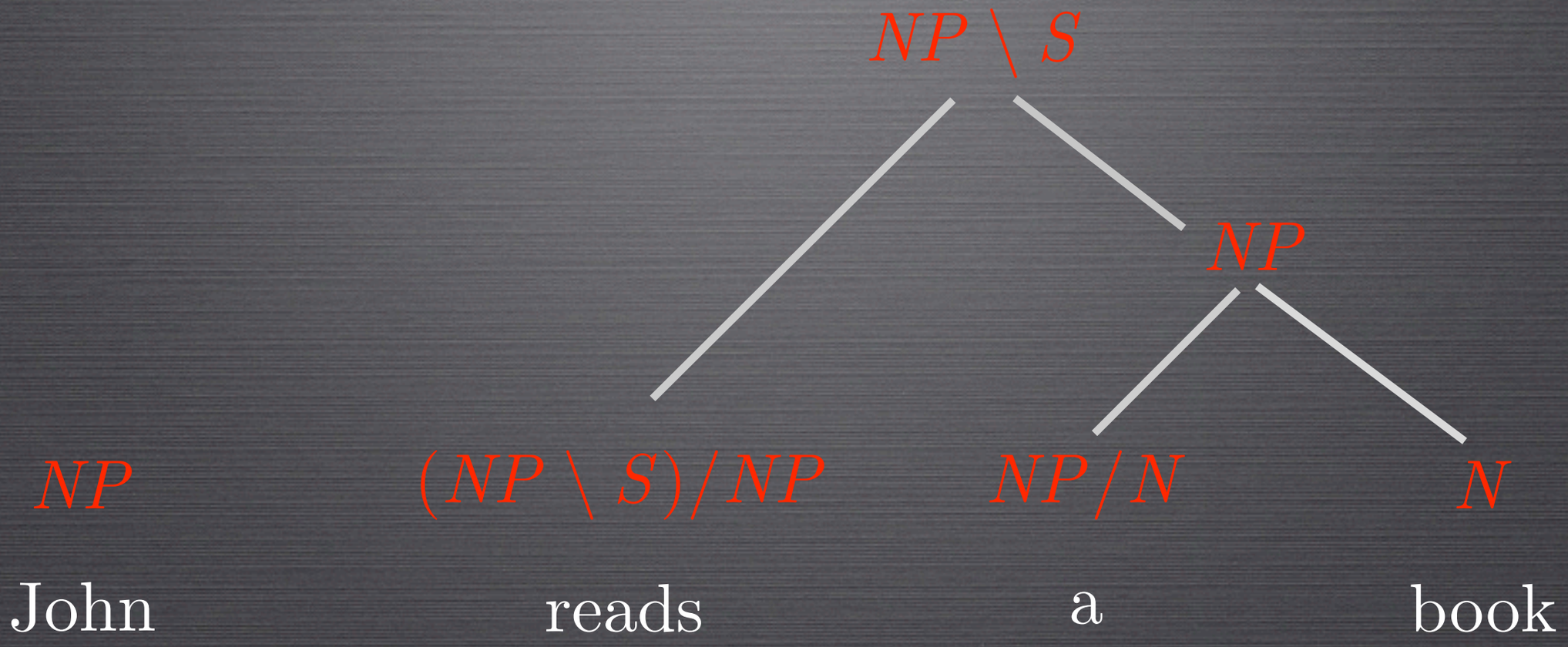
NP \ S
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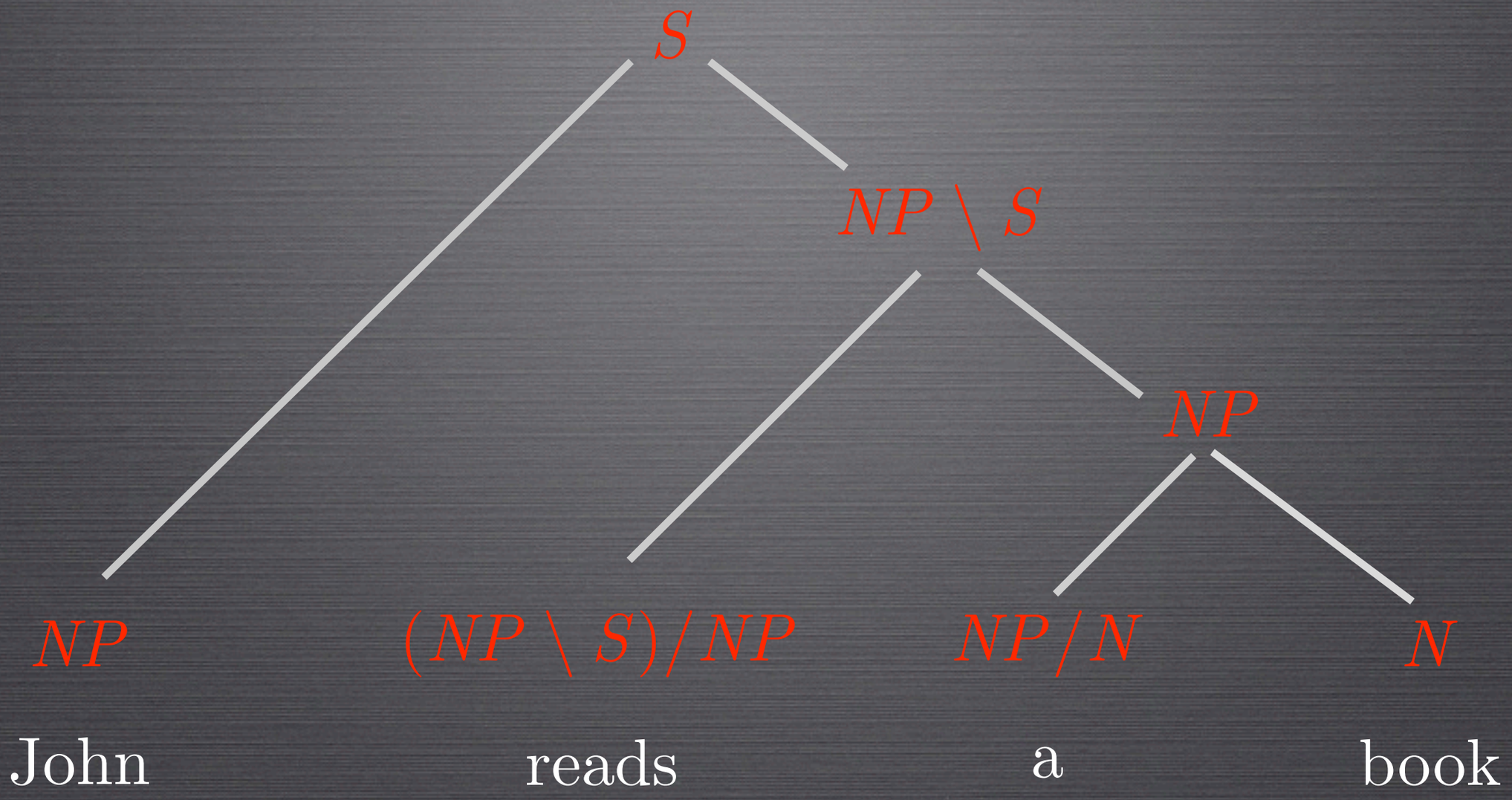
S
NP \ S
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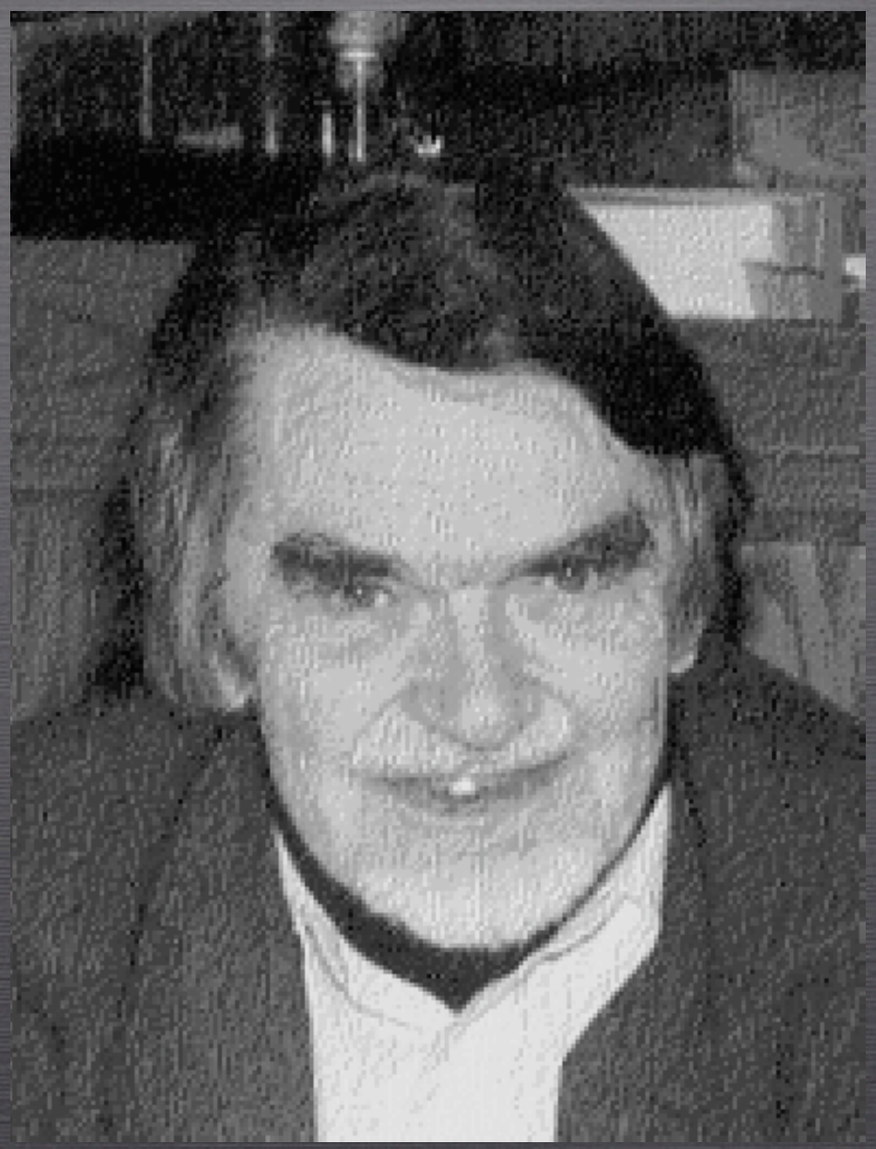
Lambek's legacy



Lambek's legacy

Functorial types, $A \setminus B$ and B / A, as implications:

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Functorial types, $A \setminus B$ and B / A, as implications:

- hypothetical reasoning
- type calculus as a substructural logic (no weakening and no contraction)
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- Syntax/semantics interface based on the Curry-Howard isomorphism

John	:	NP
loves	:	$(NP \setminus S)/NP$
Mary	:	NP
passionately	:	$S \setminus S$

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We need: $(\mathrm{NP} \setminus \mathrm{S})/\mathrm{NP} \vdash \mathrm{NP} \setminus (\mathrm{S}/\mathrm{NP})$

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loves Mary passionately : $NP \setminus S$???

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loves Mary passionately : $NP \setminus S$???

We need: $\mathrm{NP} \setminus \mathrm{S}, \mathrm{S} \setminus \mathrm{S} \vdash \mathrm{NP} \setminus \mathrm{S}$

Add the rules:

$$\textsc{if}\quad \alpha, \Gamma \vdash \beta \quad \textsc{then}\quad \Gamma \vdash \alpha \backslash \beta$$

$$\textsc{if}\quad \Gamma, \alpha \vdash \beta \quad \textsc{then}\quad \Gamma \vdash \alpha / \beta$$

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This allows/constraints the functorial types to be seen as **undirected** implications.

The logical framework approach

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Similarly, a grammatical framework should provide a means of defining various grammatical formalisms.

Architecture of grammatical systems

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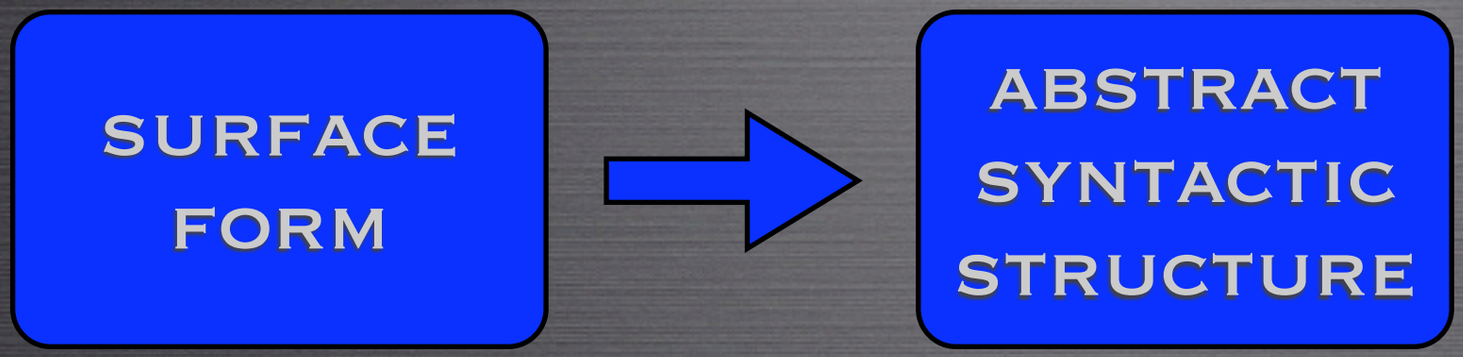
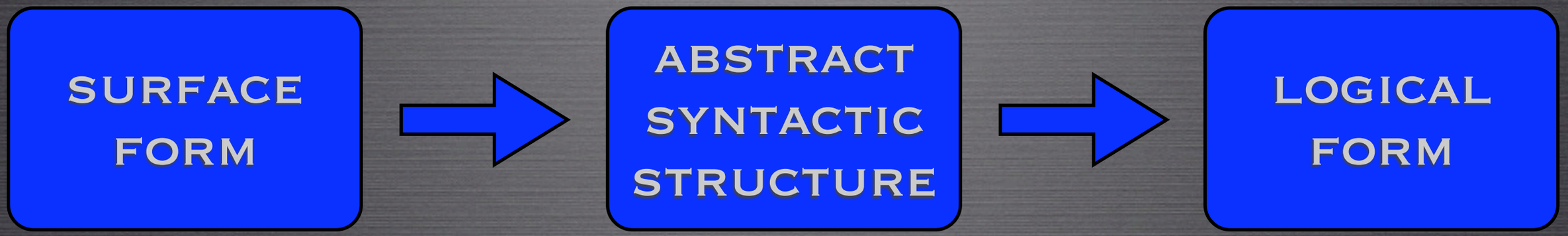
SURFACE FORM

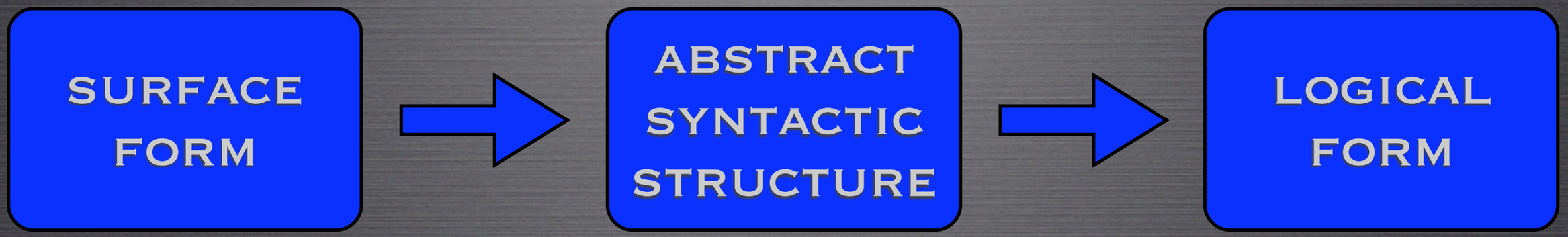
Architecture of grammatical systems



Architecture of grammatical systems

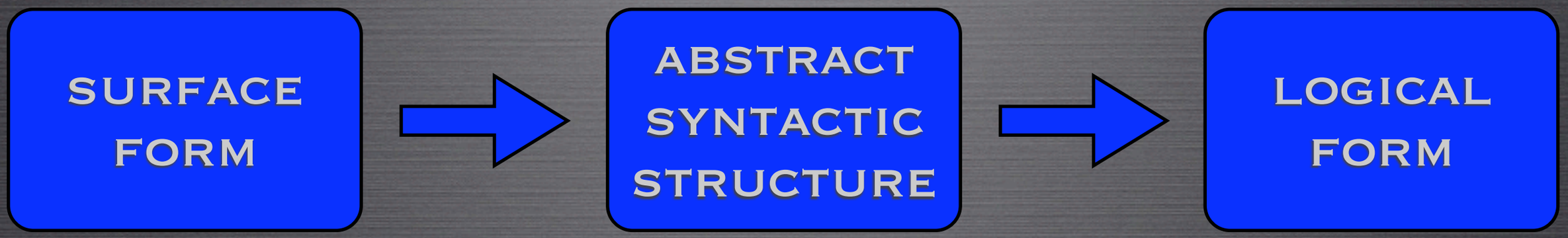


Architecture of grammatical systems



string

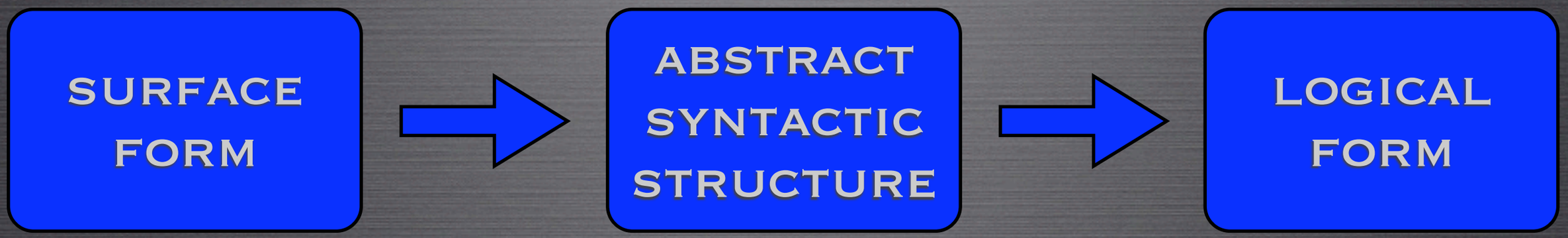
Architecture of grammatical systems



string

parse tree

Architecture of grammatical systems

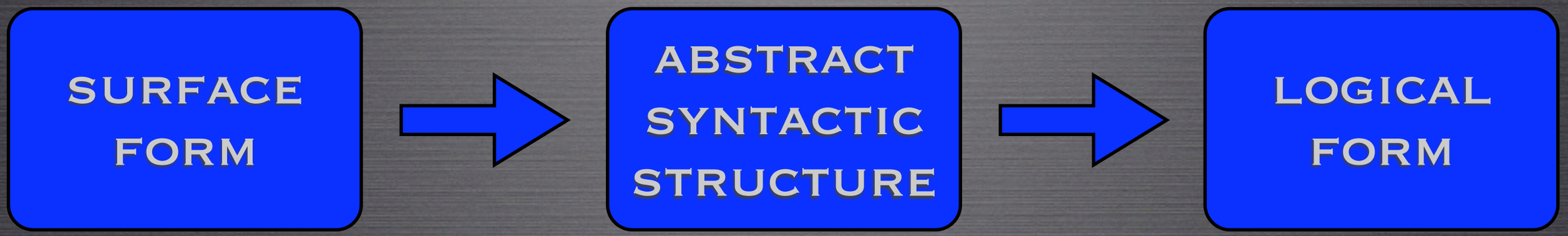


STRING

PARSE TREE

LOGICAL FORMULA

Architecture of grammatical systems



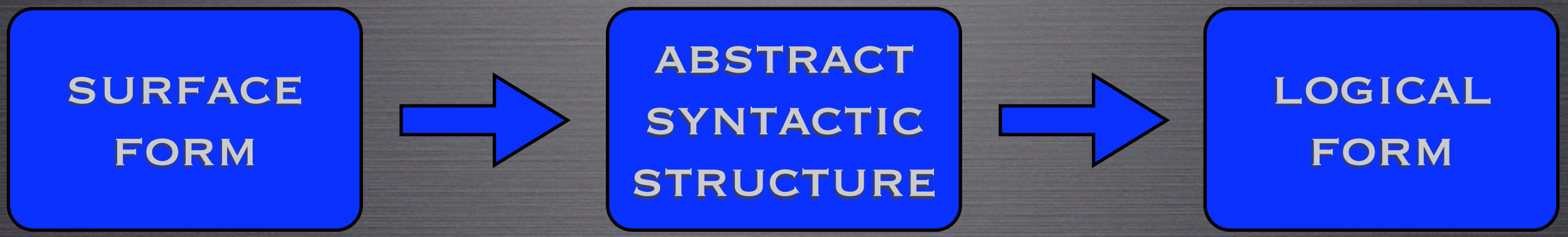
string

parse tree

logical formula

term from some prosodic algebra

Architecture of grammatical systems



string

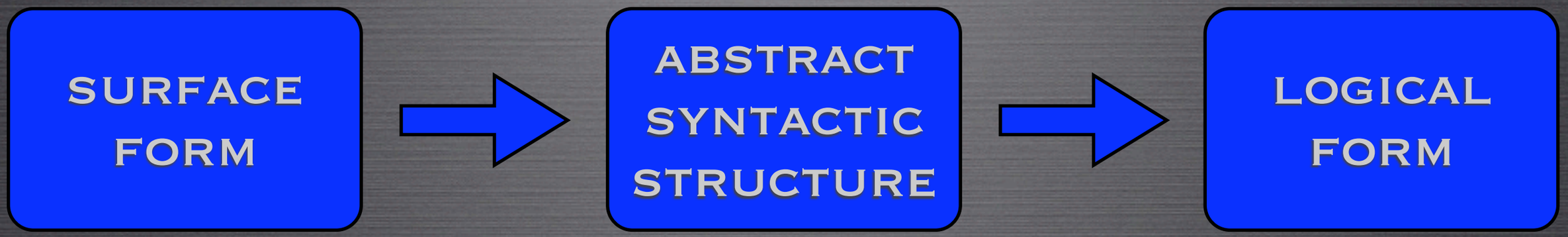
parse tree

logical formula

term from some prosodic algebra

formal proof (deduction tree)

Architecture of grammatical systems



string	parse tree	logical formula
term from some prosodic algebra	formal proof (deduction tree)	(first-order) (higher-order) (modal) (intensional)

Mathematical preliminaries

Simple types

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The set $\mathcal{T}_{\mathcal{A}}$ of simple types built upon $\mathcal{A}$ is inductively defined as follows:

$$\mathcal{T}_{\mathcal{A}} ::= \mathcal{A} \mid (\mathcal{T}_{\mathcal{A}} \to \mathcal{T}_{\mathcal{A}})$$

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We write $\mathscr{T}_\Sigma$ for $\mathscr{T}_A$

Simply typed lambda-calculus

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The set of lambda-terms built upon Σ is inductively defined as follows:

$$T ::= x \mid c \mid (\lambda x.T) \mid (T\,T)$$

where $x \in \mathcal{X}$ and $c \in C$

Typing rules:

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$$\frac{\Gamma \vdash_\Sigma t : (\alpha \to \beta) \quad \Gamma \vdash_\Sigma u : \alpha}{\Gamma \vdash_\Sigma (t\, u) : \beta} \quad \text{(app)}$$

Properties:

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A (closed) lambda-term is linear if and only if all the lambda-abstractions it contains are linear

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$$\frac{\Gamma \vdash_\Sigma t : (\alpha \to \beta) \quad \Delta \vdash_\Sigma u : \alpha}{\Gamma, \Delta \vdash_\Sigma (t\, u) : \beta} \; \mathrm{dom}(\Gamma) \cap \mathrm{dom}(\Delta) = \varnothing \quad (\text{app})$$

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We write Λ_Σ for the set of simply-typed linear lambda-terms built upon a signature Σ

Lexicons

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Given two higher-order signatures:

$$\Sigma_1 = \langle A_1, C_1, \tau_1 \rangle \quad \text{and} \quad \Sigma_2 = \langle A_2, C_2, \tau_2 \rangle$$

A lexicon $\mathscr{L} : \Sigma_1 \to \Sigma_2$ is a pair $\mathscr{L} = \langle \eta, \theta \rangle$ such that:

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$$\theta : C_1 \to \Lambda_{\Sigma_2}$$

$$\vdash_{\Sigma_2} \theta(c) : \eta(\tau_1(c))$$


typing
Λ_{Σ_1}
$\mathscr{T}_{A_1}$


typing
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typing
Λ_{Σ_2}
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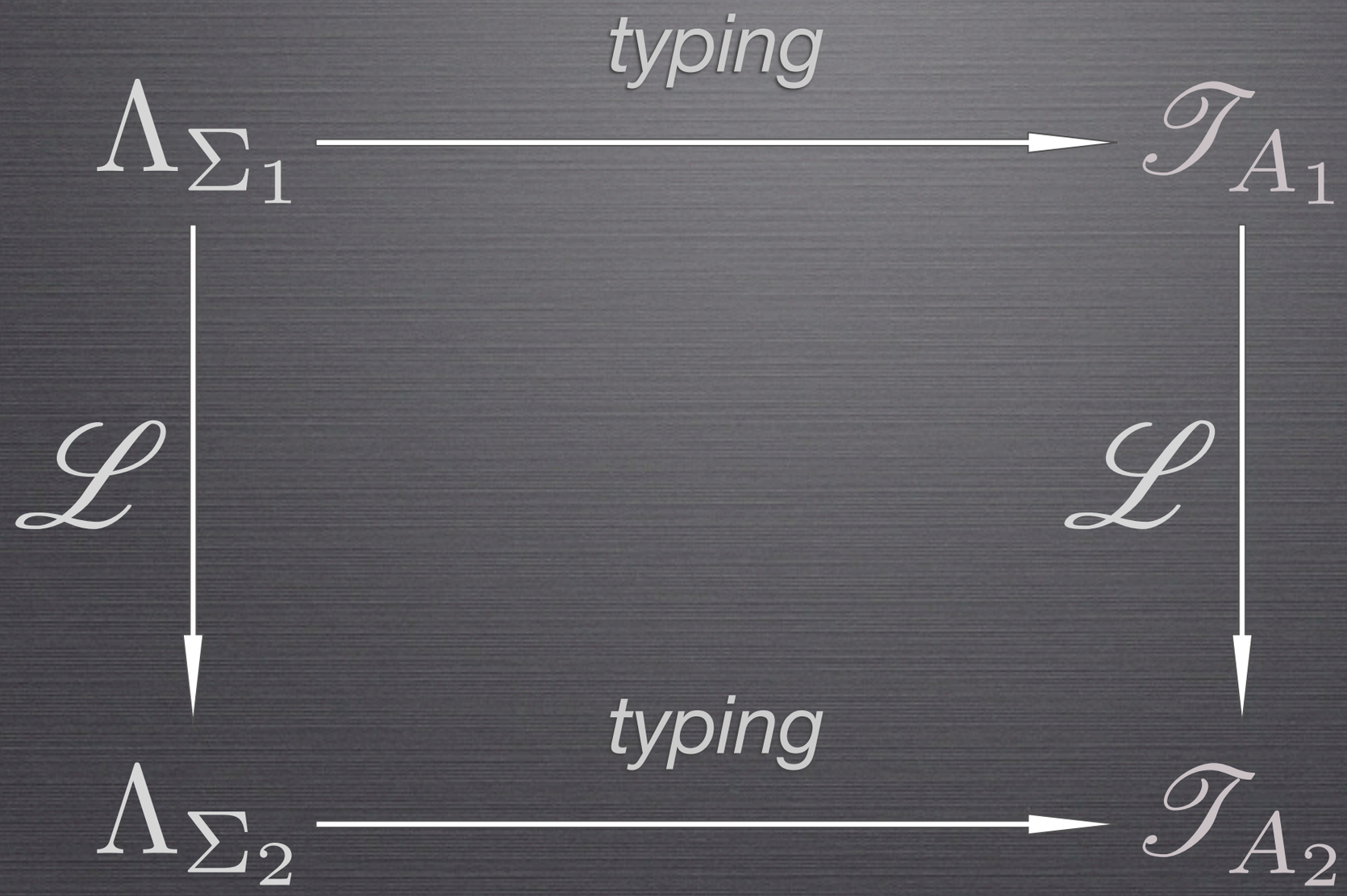
typing
Λ_{Σ_1}
$\mathscr{T}_{A_1}$
$\mathscr{L}$
$\mathscr{L}$
typing
Λ_{Σ_2}
$\mathscr{T}_{A_2}$