

INTRODUCTION TO
ABSTRACT CATEGORIAL
GRAMMARS

FIFTH COURSE

A GENERAL PARSING ALGORITHM

AN EXAMPLE:

AN EXAMPLE:

RETURN OF THE UNICORN

N, NP, S : type
J : NP
U : N
A : $N \rightarrow ((NP \rightarrow S) \rightarrow S)$
S : $((NP \rightarrow S) \rightarrow S) \rightarrow (NP \rightarrow S)$

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/a/, /John/, /seeks/, /unicorn/ : $\sigma \rightarrow \sigma$

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N, NP, S := $\sigma \rightarrow \sigma$
J := /John/
U := /unicorn/
A := $\lambda x. \lambda p. p (/a/ + x)$
S := $\lambda p. \lambda x. p (\lambda y. x + /seeks/ + y)$

σ : type
/a/, /John/, /seeks/, /unicorn/ : $\sigma \rightarrow \sigma$

GENERATE AND TEST ALGORITHM

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1. TRY TO PROVE S USING THE TYPES OF THE ABSTRACT CONSTANTS AS PROPER AXIOMS:

I.E., USING: NP

N

$N \rightarrow ((NP \rightarrow S) \rightarrow S)$

$((NP \rightarrow S) \rightarrow S) \rightarrow (NP \rightarrow S)$

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2. BY THE CURRY-HOWARD ISOMORPHISM, YOU HAVE CONSTRUCTED A TERM OF THE ABSTRACT LANGUAGE. APPLY THE LEXICON TO THIS TERM.

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2. BY THE CURRY-HOWARD ISOMORPHISM, YOU HAVE CONSTRUCTED A TERM OF THE ABSTRACT LANGUAGE. APPLY THE LEXICON TO THIS TERM.
3. CHECK WHETHER THE RESULTING OBJECT TERM IS EQUAL TO THE TERM YOU HAVE TO PARSE.

GETTING RID OF OBJECT TERMS

GETTING RID OF OBJECT TERMS

THE COHERENCE THEOREM

STRIKES BACK

GETTING RID OF OBJECT TERMS

THE COHERENCE THEOREM STRIKES BACK

SPECIALIZE YOUR OBJECT SIGNATURE IN SUCH A
WAY THAT THE TERM TO PARSE IS COMPLETELY
DETERMINED BY ITS TYPE:

GETTING RID OF OBJECT TERMS

THE COHERENCE THEOREM STRIKES BACK

SPECIALIZE YOUR OBJECT SIGNATURE IN SUCH A WAY THAT THE TERM TO PARSE IS COMPLETELY DETERMINED BY ITS TYPE:

$$\begin{array}{ll} \sigma_0, \sigma_1, \sigma_2, \sigma_3, \sigma_4 & : \text{ type} \\ \quad \quad \quad \text{/a/} & : \sigma_1 \rightarrow \sigma_2 \\ \quad \quad \quad \text{/John/} & : \sigma_3 \rightarrow \sigma_4 \\ \quad \quad \quad \text{/seeks/} & : \sigma_2 \rightarrow \sigma_3 \\ \quad \quad \quad \text{/unicorn/} & : \sigma_0 \rightarrow \sigma_1 \end{array}$$

GETTING RID OF OBJECT TERMS

THE COHERENCE THEOREM STRIKES BACK

SPECIALIZE YOUR OBJECT SIGNATURE IN SUCH A WAY THAT THE TERM TO PARSE IS COMPLETELY DETERMINED BY ITS TYPE:

$$\begin{array}{lcl} \sigma_0, \sigma_1, \sigma_2, \sigma_3, \sigma_4 & : & \textit{type} \\ /a/ & : & \sigma_1 \rightarrow \sigma_2 \\ /John/ & : & \sigma_3 \rightarrow \sigma_4 \\ /seeks/ & : & \sigma_2 \rightarrow \sigma_3 \\ /unicorn/ & : & \sigma_0 \rightarrow \sigma_1 \end{array}$$

$$\vdash /John/ + /seeks/ + /a/ + /unicorn/ : \sigma_0 \rightarrow \sigma_4$$

**SPECIALIZE THE LEXICON ACCORDINGLY AND
COMPUTE ALL THE POSSIBLE TYPINGS OF THE
LEXICAL ENTRIES:**

REVENGE OF THE PRINCIPAL TYPE

SPECIALIZE THE LEXICON ACCORDINGLY AND
COMPUTE ALL THE POSSIBLE TYPINGS OF THE
LEXICAL ENTRIES:

$$\begin{array}{lcl} J & := & /John/ & : & \sigma_3 \rightarrow \sigma_4 \\ U & := & /unicorn/ & : & \sigma_0 \rightarrow \sigma_1 \\ A & := & \lambda x. \lambda p. p (/a/ + x) & : & \\ & & (\sigma_i \rightarrow \sigma_1) \rightarrow & & \\ & & (((\sigma_i \rightarrow \sigma_2) \rightarrow (\sigma_j \rightarrow \sigma_k)) \rightarrow (\sigma_j \rightarrow \sigma_k)) & & \\ S & := & \lambda p. \lambda x. p (\lambda y. x + /seeks/ + y) & : & \\ & & (((\sigma_l \rightarrow \sigma_2) \rightarrow (\sigma_l \rightarrow \sigma_m)) \rightarrow (\sigma_n \rightarrow \sigma_p)) \rightarrow & & \\ & & ((\sigma_3 \rightarrow \sigma_m) \rightarrow (\sigma_n \rightarrow \sigma_p)) & & \end{array}$$

TRY TO PROVE: $\langle S, \sigma_0 \rightarrow \sigma_4 \rangle$

TRY TO PROVE: $\langle S, \sigma_0 \rightarrow \sigma_4 \rangle$, USING:

$$\langle NP, \sigma_3 \rightarrow \sigma_4 \rangle$$

$$\langle N, \sigma_0 \rightarrow \sigma_1 \rangle$$

$$\langle N \rightarrow (NP \rightarrow S) \rightarrow S,$$

$$(\sigma_i \rightarrow \sigma_1) \rightarrow$$

$$((\sigma_i \rightarrow \sigma_2) \rightarrow (\sigma_j \rightarrow \sigma_k)) \rightarrow$$

$$(\sigma_j \rightarrow \sigma_k) \rangle$$

$$\langle ((NP \rightarrow S) \rightarrow S) \rightarrow NP \rightarrow S,$$

$$(((\sigma_l \rightarrow \sigma_2) \rightarrow (\sigma_l \rightarrow \sigma_m)) \rightarrow (\sigma_n \rightarrow \sigma_p)) \rightarrow$$

$$(\sigma_3 \rightarrow \sigma_m) \rightarrow$$

$$(\sigma_n \rightarrow \sigma_p) \rangle$$

ELIMINATING REDUNDANCIES

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A NEW HOPE

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A NEW HOPE

CONSIDER THE FOLLOWING PAIR:

$$\langle N \rightarrow (NP \rightarrow S) \rightarrow S, \\ (\sigma_i \rightarrow \sigma_1) \rightarrow ((\sigma_i \rightarrow \sigma_2) \rightarrow (\sigma_j \rightarrow \sigma_k)) \rightarrow (\sigma_j \rightarrow \sigma_k) \rangle$$

ELIMINATING REDUNDANCIES

A NEW HOPE

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THE SHAPE OF THE SPECIALIZED OBJECT TYPE IS COMPLETELY SPECIFIED BY THE GRAMMAR. THE ONLY RELEVANT INFORMATION IS GIVEN BY THE INDICES.

ELIMINATING REDUNDANCIES

A NEW HOPE

CONSIDER THE FOLLOWING PAIR:

$$\langle N \rightarrow (NP \rightarrow S) \rightarrow S, \\ (\sigma_i \rightarrow \sigma_1) \rightarrow ((\sigma_i \rightarrow \sigma_2) \rightarrow (\sigma_j \rightarrow \sigma_k)) \rightarrow (\sigma_j \rightarrow \sigma_k) \rangle$$

THE SHAPE OF THE SPECIALIZED OBJECT TYPE IS COMPLETELY SPECIFIED BY THE GRAMMAR. THE ONLY RELEVANT INFORMATION IS GIVEN BY THE INDICES.

REPLACE THE ABOVE PAIR BY THE FOLLOWING FORMULA:

$$N[i, 1] \rightarrow (NP[i, 2] \rightarrow S[j, k]) \rightarrow S[j, k]$$

WE END UP WITH THE FOLLOWING PROBLEM:

WE END UP WITH THE FOLLOWING PROBLEM:

PROVE:

$$S[0, 4]$$

WE END UP WITH THE FOLLOWING PROBLEM:

PROVE:

$$S[0, 4]$$

USING:

$$J : NP[3, 4]$$

$$U : N[0, 1]$$

$$A : N[i, 1] \rightarrow (NP[i, 2] \rightarrow S[j, k]) \rightarrow S[j, k]$$

$$S : ((NP[l, 2] \rightarrow S[l, m]) \rightarrow S[n, p]) \rightarrow NP[3, m] \rightarrow S[n, p]$$

$$A : N[i, 1] \rightarrow (NP[i, 2] \rightarrow S[j, k]) \rightarrow S[j, k]$$

$$U : N[0, 1]$$

$A : N[i, 1] \rightarrow (NP[i, 2] \rightarrow S[j, k]) \rightarrow S[j, k]$

$U : N[0, 1]$

$A U : (NP[0, 2] \rightarrow S[j, k]) \rightarrow S[j, k]$

$A : N[i, 1] \rightarrow (NP[i, 2] \rightarrow S[j, k]) \rightarrow S[j, k]$

$U : N[0, 1]$

$A U : (NP[0, 2] \rightarrow S[j, k]) \rightarrow S[j, k]$

$S : ((NP[l, 2] \rightarrow S[l, m]) \rightarrow S[n, p]) \rightarrow NP[3, m] \rightarrow S[n, p]$

$A : N[i, 1] \rightarrow (NP[i, 2] \rightarrow S[j, k]) \rightarrow S[j, k]$

$U : N[0, 1]$

$A U : (NP[0, 2] \rightarrow S[j, k]) \rightarrow S[j, k]$

$S : ((NP[l, 2] \rightarrow S[l, m]) \rightarrow S[n, p]) \rightarrow NP[3, m] \rightarrow S[n, p]$

$S(A U) : NP[3, m] \rightarrow S[0, m]$

$A : N[i, 1] \rightarrow (NP[i, 2] \rightarrow S[j, k]) \rightarrow S[j, k]$

$U : N[0, 1]$

$AU : (NP[0, 2] \rightarrow S[j, k]) \rightarrow S[j, k]$

$S : ((NP[l, 2] \rightarrow S[l, m]) \rightarrow S[n, p]) \rightarrow NP[3, m] \rightarrow S[n, p]$

$S(AU) : NP[3, m] \rightarrow S[0, m]$

$J : NP[3, 4]$

$A : N[i, 1] \rightarrow (NP[i, 2] \rightarrow S[j, k]) \rightarrow S[j, k]$

$U : N[0, 1]$

$AU : (NP[0, 2] \rightarrow S[j, k]) \rightarrow S[j, k]$

$S : ((NP[l, 2] \rightarrow S[l, m]) \rightarrow S[n, p]) \rightarrow NP[3, m] \rightarrow S[n, p]$

$S(AU) : NP[3, m] \rightarrow S[0, m]$

$J : NP[3, 4]$

$S(AU)J : S[0, 4]$

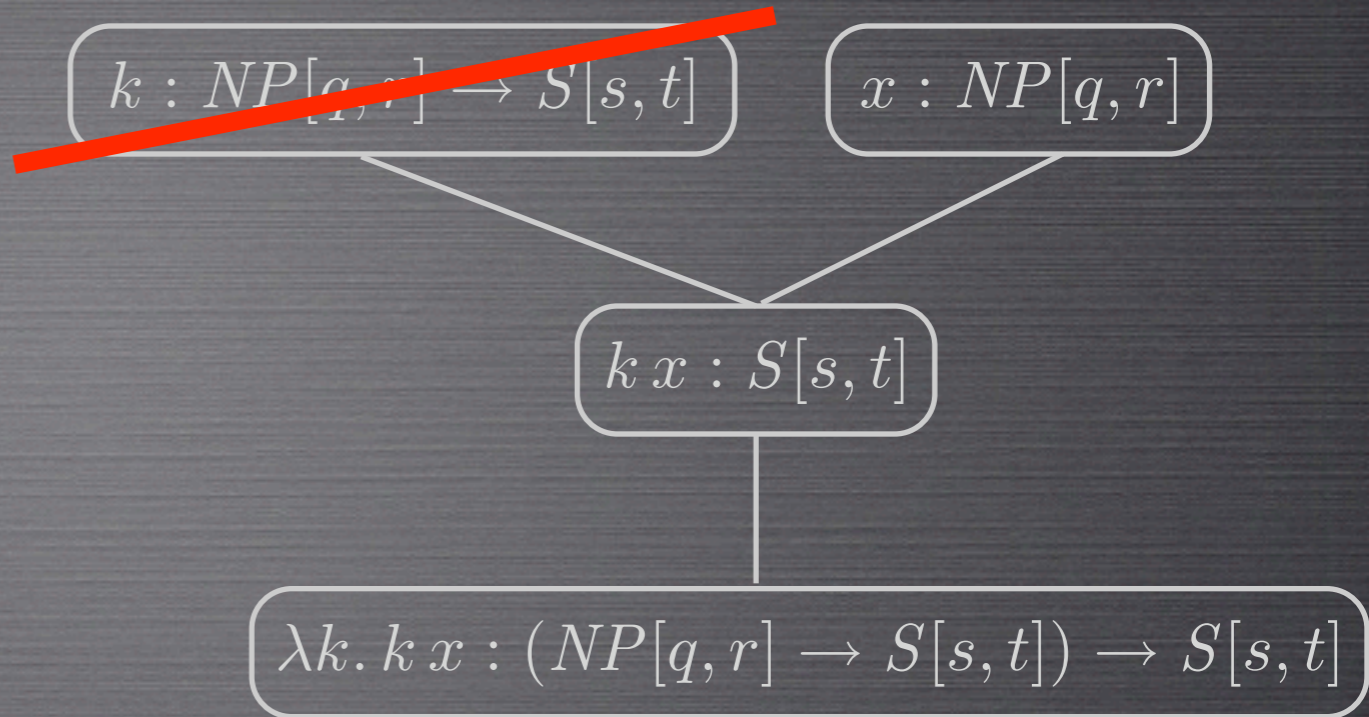
$$k : NP[q, r] \rightarrow S[s, t]$$

$$x : NP[q, r]$$

$k : NP[q, r] \rightarrow S[s, t]$

$x : NP[q, r]$

$kx : S[s, t]$



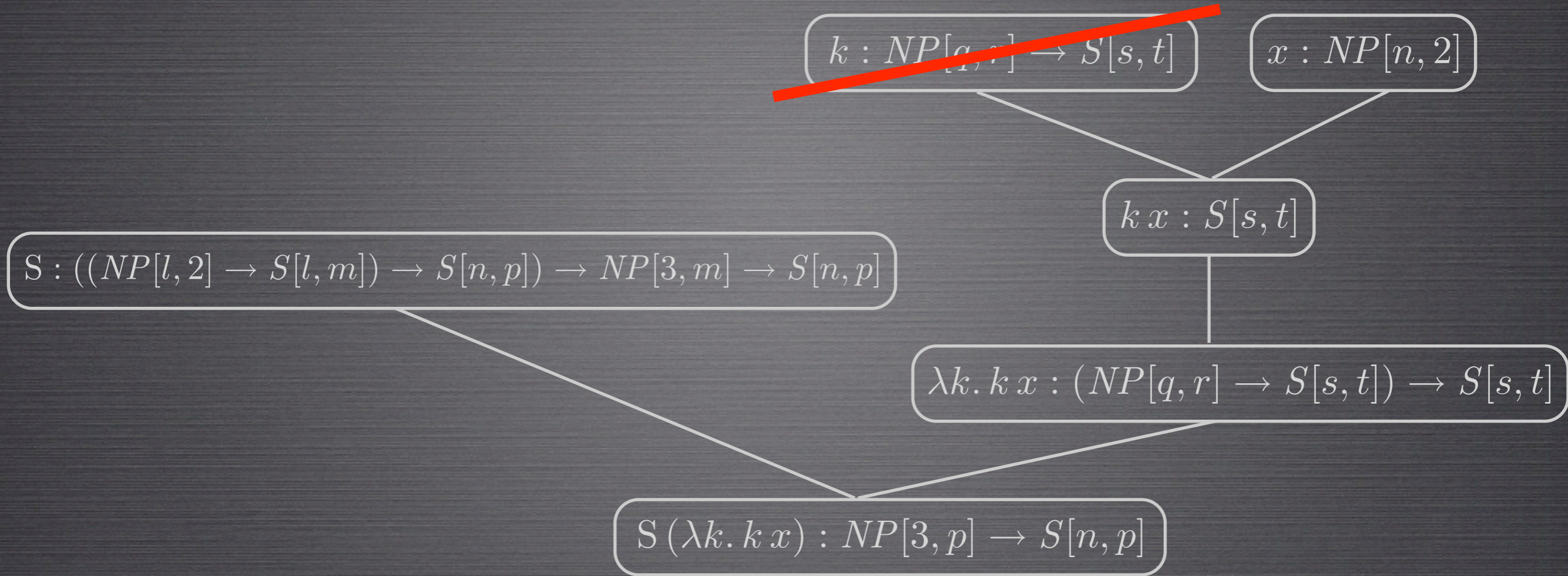
~~$k : NP[q, r] \rightarrow S[s, t]$~~

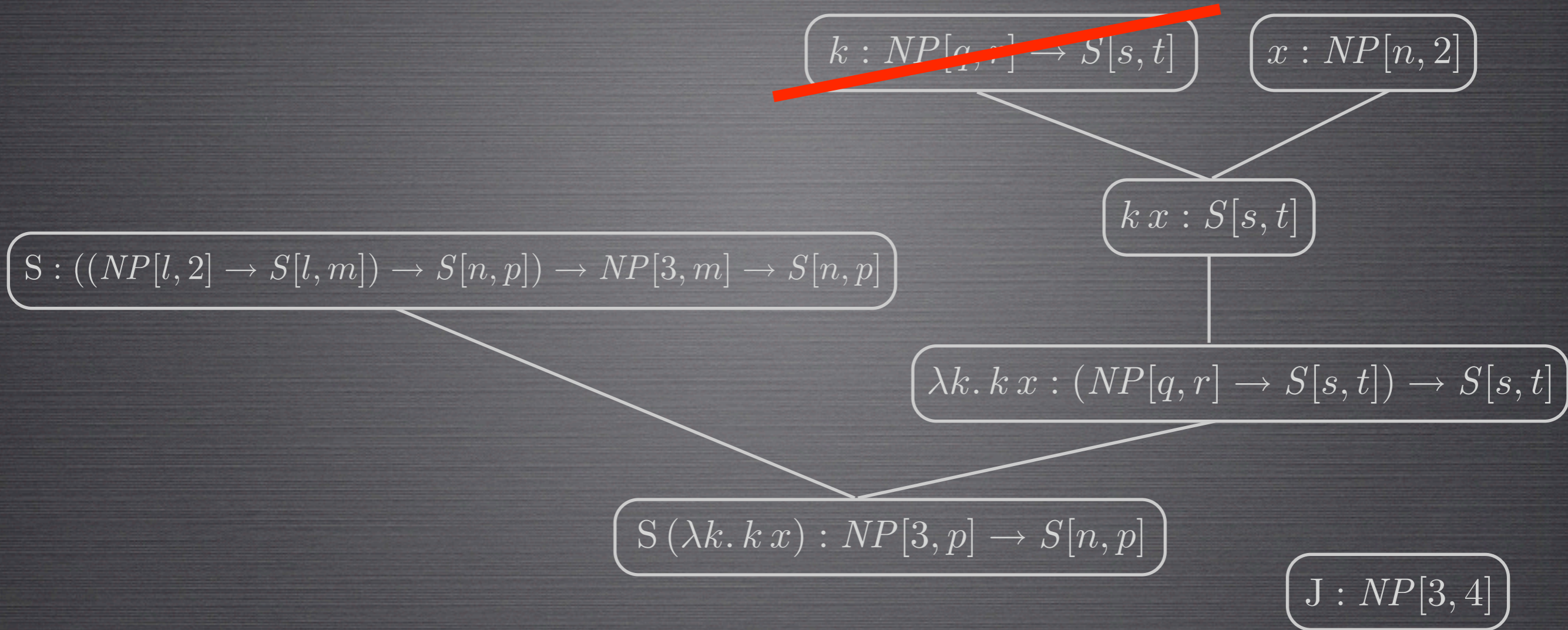
$x : NP[q, r]$

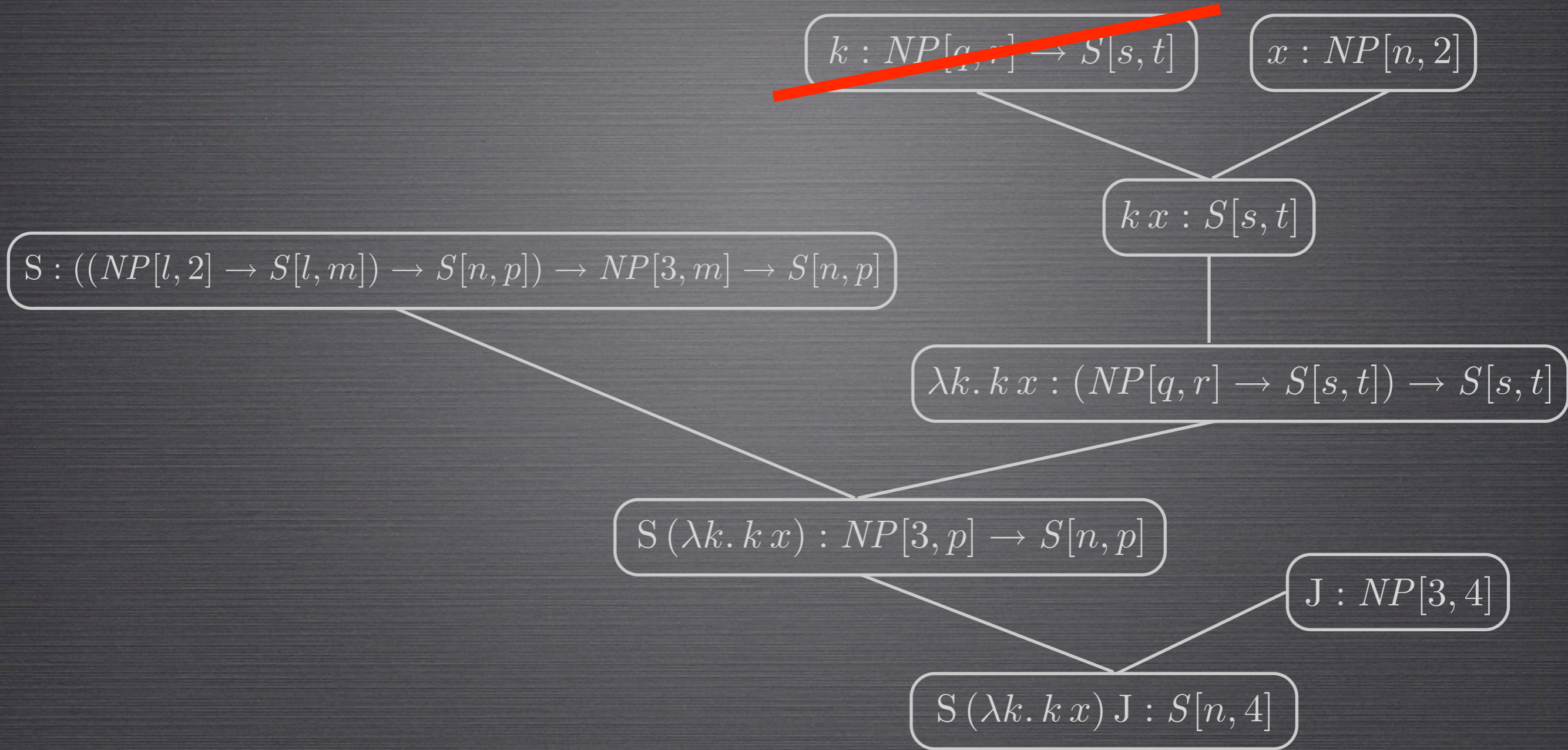
$k x : S[s, t]$

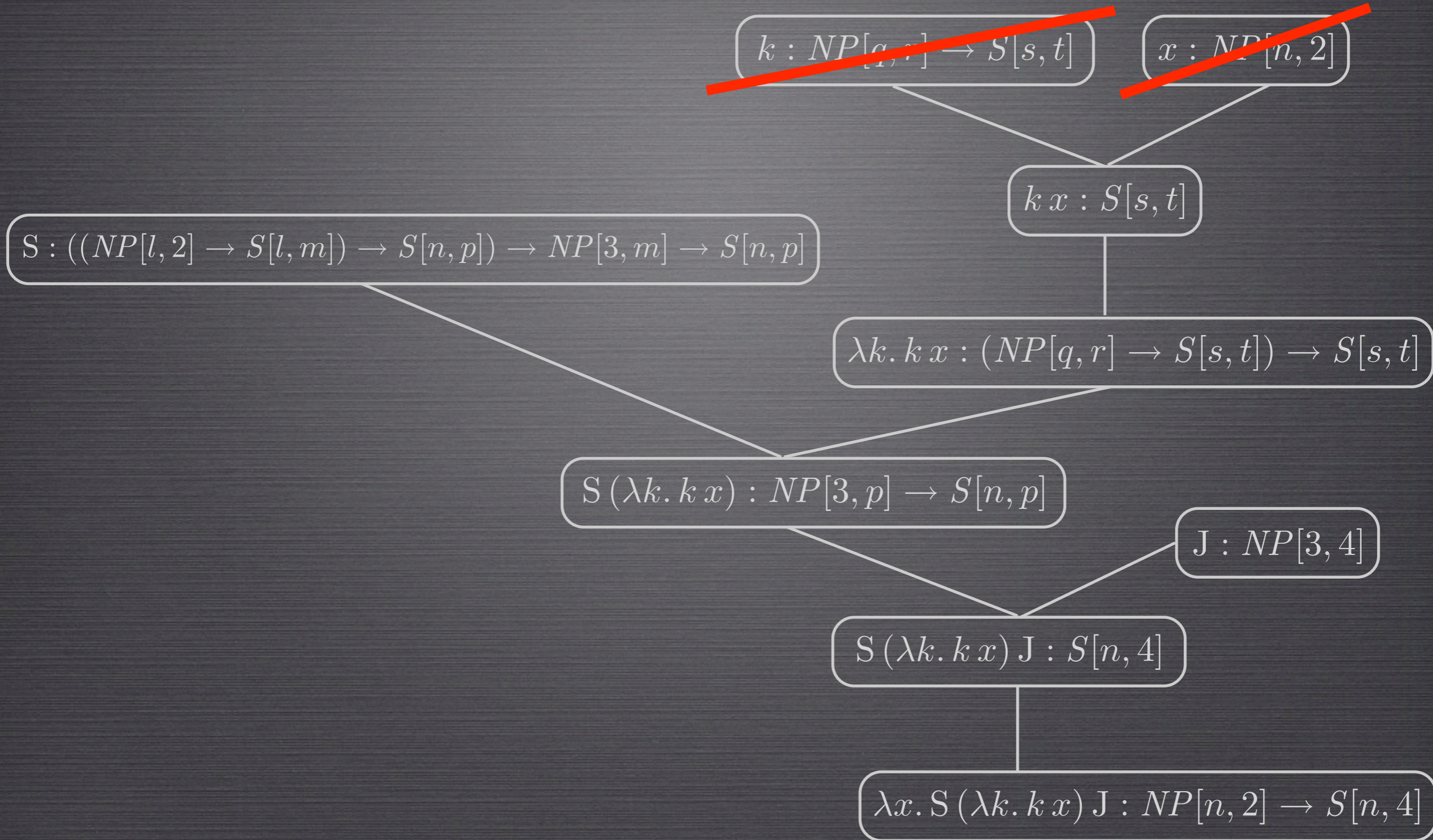
$S : ((NP[l, 2] \rightarrow S[l, m]) \rightarrow S[n, p]) \rightarrow NP[3, m] \rightarrow S[n, p]$

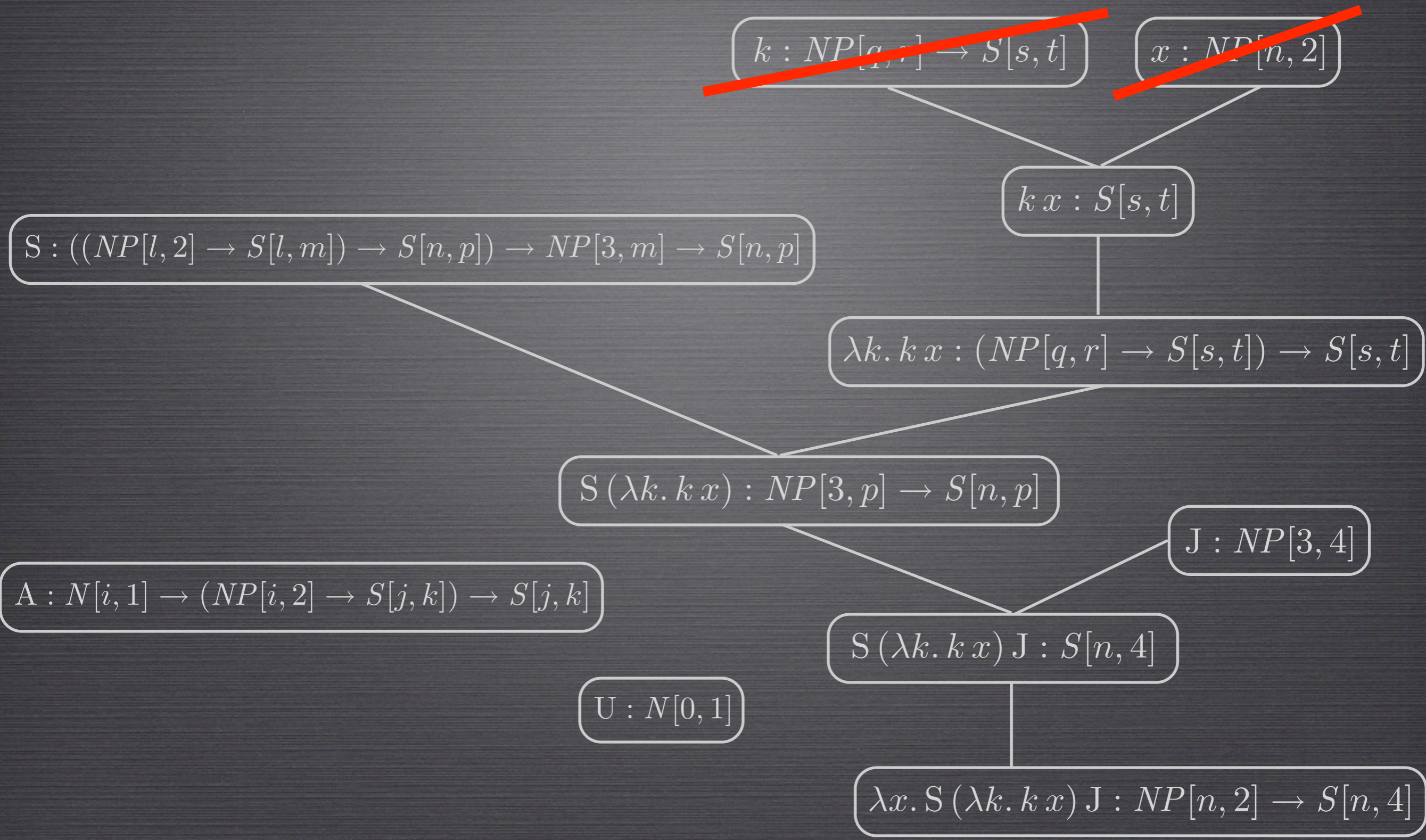
$\lambda k. k x : (NP[q, r] \rightarrow S[s, t]) \rightarrow S[s, t]$

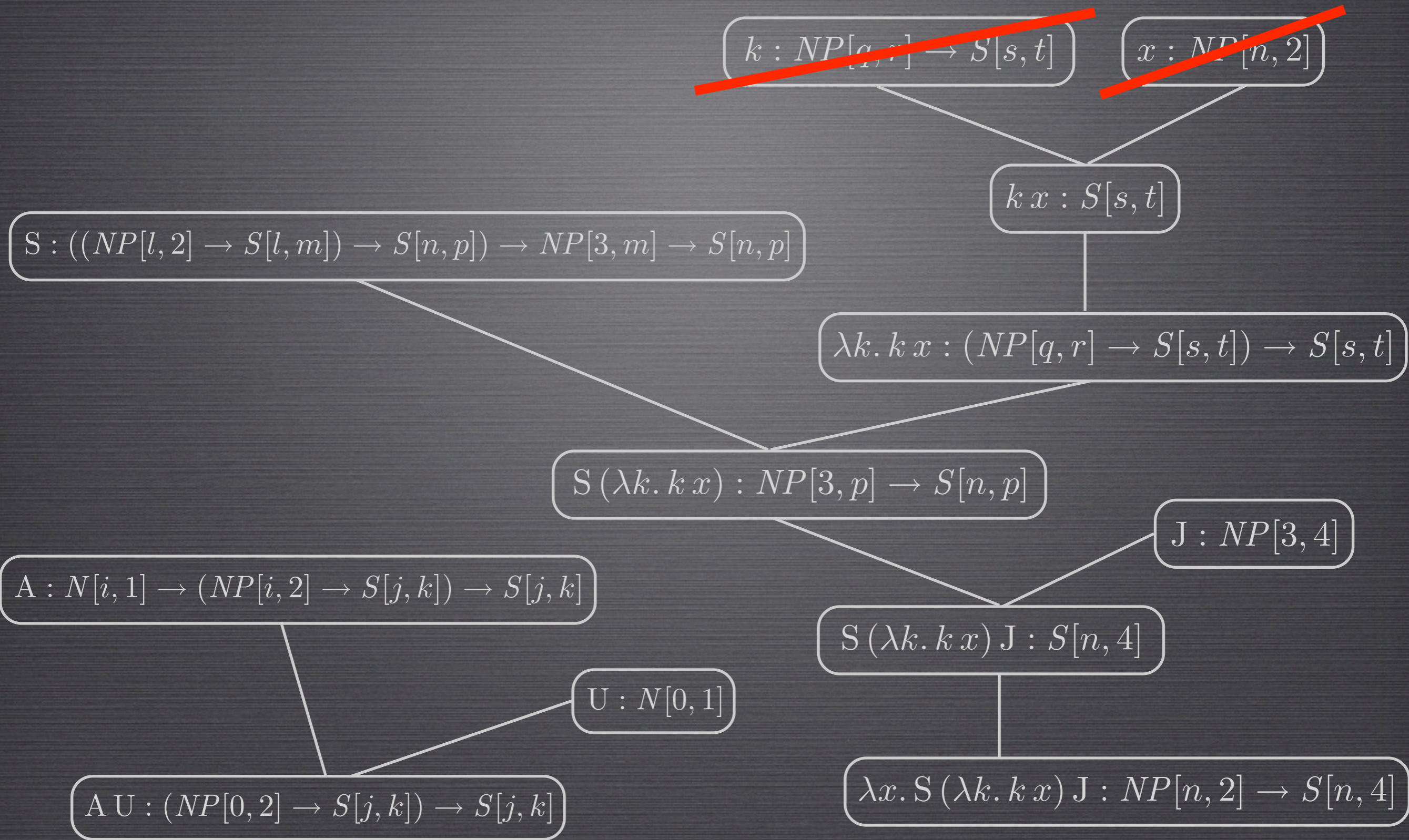












~~$k : NP[q, r] \rightarrow S[s, t]$~~

~~$x : NP[n, 2]$~~

$k x : S[s, t]$

$S : ((NP[l, 2] \rightarrow S[l, m]) \rightarrow S[n, p]) \rightarrow NP[3, m] \rightarrow S[n, p]$

$\lambda k. k x : (NP[q, r] \rightarrow S[s, t]) \rightarrow S[s, t]$

$S (\lambda k. k x) : NP[3, p] \rightarrow S[n, p]$

$J : NP[3, 4]$

$A : N[i, 1] \rightarrow (NP[i, 2] \rightarrow S[j, k]) \rightarrow S[j, k]$

$S (\lambda k. k x) J : S[n, 4]$

$U : N[0, 1]$

$\lambda x. S (\lambda k. k x) J : NP[n, 2] \rightarrow S[n, 4]$

$A U : (NP[0, 2] \rightarrow S[j, k]) \rightarrow S[j, k]$

$A U (\lambda x. S (\lambda k. k x) J) : S[0, 4]$

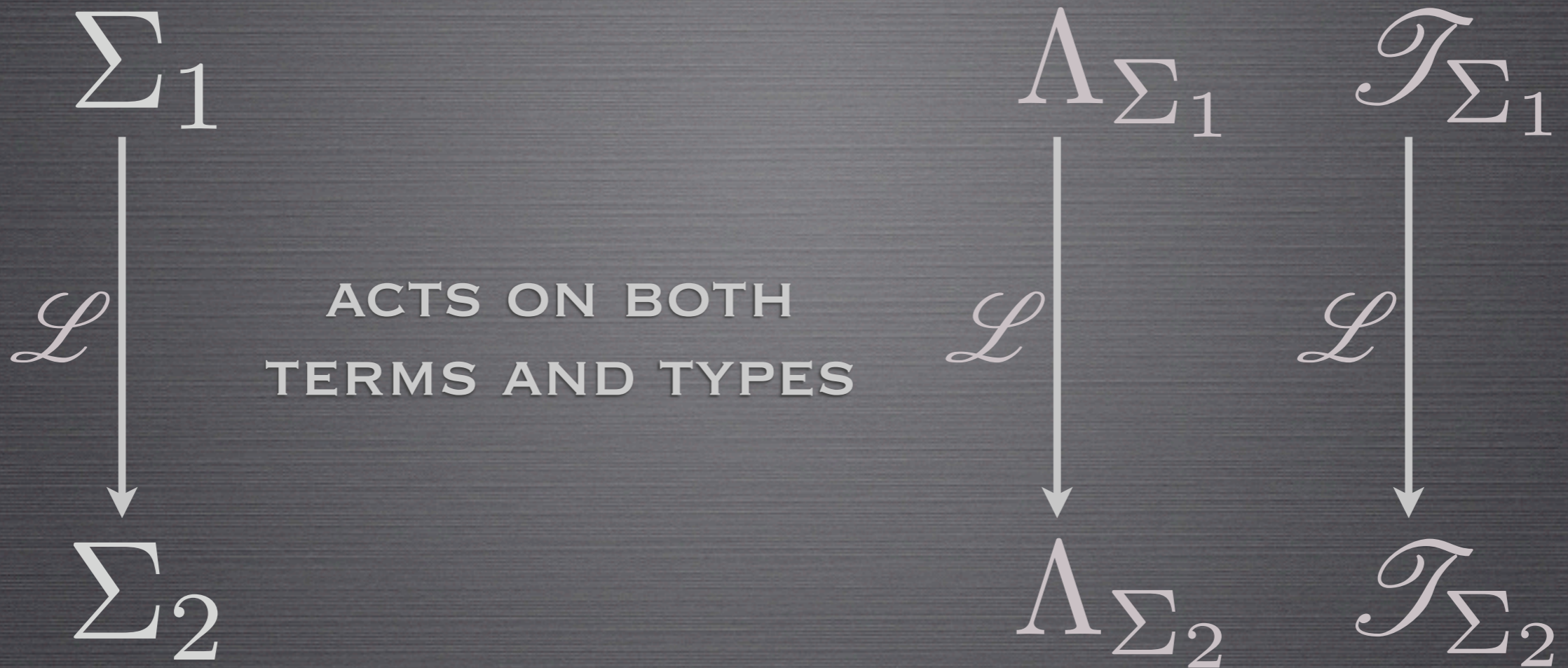
**ABSTRACT
CATEGORIAL
ARCHITECTURE**

Σ_1  Σ_2



ACTS ON BOTH
TERMS AND TYPES





SUCH THAT:

IF $\Gamma \vdash_{\Sigma_1} t : \alpha$ THEN $\mathcal{L}(\Gamma) \vdash_{\Sigma_2} \mathcal{L}(t) : \mathcal{L}(\alpha)$

TYPE-THEORETIC EXTENSIONS

CARTESIAN PRODUCT, DISJOINT UNION, AND UNIT

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$$\frac{\Gamma \vdash \alpha \quad \Gamma \vdash \beta}{\Gamma \vdash \alpha \times \beta}$$

$$\frac{\Gamma, \alpha \vdash \gamma}{\Gamma, \alpha \times \beta \vdash \gamma}$$

$$\frac{\Gamma, \beta \vdash \gamma}{\Gamma, \alpha \times \beta \vdash \gamma}$$

CARTESIAN PRODUCT, DISJOINT UNION, AND UNIT

$$\frac{\Gamma \vdash \alpha \quad \Gamma \vdash \beta}{\Gamma \vdash \alpha \times \beta}$$

$$\frac{\Gamma, \alpha \vdash \gamma}{\Gamma, \alpha \times \beta \vdash \gamma}$$

$$\frac{\Gamma, \beta \vdash \gamma}{\Gamma, \alpha \times \beta \vdash \gamma}$$

$$\frac{\Gamma \vdash \beta}{\Gamma \vdash \alpha + \beta}$$

$$\frac{\Gamma \vdash \alpha}{\Gamma \vdash \alpha + \beta}$$

$$\frac{\Gamma, \alpha \vdash \gamma \quad \Gamma, \beta \vdash \gamma}{\Gamma, \alpha + \beta \vdash \gamma}$$

CARTESIAN PRODUCT, DISJOINT UNION, AND UNIT

$$\frac{\Gamma \vdash \alpha \quad \Gamma \vdash \beta}{\Gamma \vdash \alpha \times \beta}$$

$$\frac{\Gamma, \alpha \vdash \gamma}{\Gamma, \alpha \times \beta \vdash \gamma}$$

$$\frac{\Gamma, \beta \vdash \gamma}{\Gamma, \alpha \times \beta \vdash \gamma}$$

$$\frac{\Gamma \vdash \beta}{\Gamma \vdash \alpha + \beta}$$

$$\frac{\Gamma \vdash \alpha}{\Gamma \vdash \alpha + \beta}$$

$$\frac{\Gamma, \alpha \vdash \gamma \quad \Gamma, \beta \vdash \gamma}{\Gamma, \alpha + \beta \vdash \gamma}$$

$$\vdash 1$$

$$\frac{\Gamma \vdash \alpha}{\Gamma, 1 \vdash \alpha}$$

VARIANTS

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NP, S : type
 $opt_NP = \{\text{none} \mid \text{some of } NP\}$: type
READ : $opt_NP \rightarrow NP \rightarrow S$

VARIANTS

$$\begin{array}{lcl} NP, S & : & \text{type} \\ opt_NP = \{\text{none} \mid \text{some of } NP\} & : & \text{type} \\ \text{READ} & : & opt_NP \rightarrow NP \rightarrow S \end{array}$$
$$\text{READ} := \lambda xy. y + \text{/reads/} + \left(\begin{array}{lcl} \text{case } x & \text{of} & \text{none} \quad \rightarrow \quad \epsilon \\ & & \mid \quad (\text{some } z) \quad \rightarrow \quad z \end{array} \right)$$

RECORDS

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$$\left[\begin{array}{l} \text{inflection} \\ \text{function} \end{array} \right] = \left[\begin{array}{l} \text{gender} = \text{masc} \\ \text{number} = \text{sing} \\ \text{object (indirect (a_Prep))} \end{array} \right]$$

RECORDS

$$\left[\begin{array}{l} \text{inflection} \\ \text{function} \end{array} = \left[\begin{array}{l} \text{gender} = \text{masc} \\ \text{number} = \text{sing} \end{array} \right] \right]$$

= object (indirect (a_Prep))

Gender = {masc | fem} : type
Number = {sing | plur} : type
Inflection = [gender : *Gender*; number : *Number*] : type
Preposition = {a_Prep | de_Prep} : type
Direction = {direct | indirect of *Preposition*} : type
Function = {subject | object of *Direction*} : type
Features = [inflection : *Inflection*; function : *Function*] : type

NON LINEARITY

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$\Gamma; x : \alpha \vdash x : \alpha$ (lin. var.)

$\Gamma, x : \alpha; \vdash x : \alpha$ (var.)

NON LINEARITY

$\Gamma; x : \alpha \vdash x : \alpha$ (lin. var.)

$\Gamma, x : \alpha; \vdash x : \alpha$ (var.)

$$\frac{\Gamma; \Delta, x : \alpha \vdash t : \beta}{\Gamma; \Delta \vdash \lambda^{\circ} x. t : \alpha \multimap \beta} \quad (\text{lin. abs.})$$

NON LINEARITY

$\Gamma; x : \alpha \vdash x : \alpha$ (lin. var.)

$\Gamma, x : \alpha; \vdash x : \alpha$ (var.)

$$\frac{\Gamma; \Delta, x : \alpha \vdash t : \beta}{\Gamma; \Delta \vdash \lambda^\circ x. t : \alpha \multimap \beta} \quad (\text{lin. abs.})$$
$$\frac{\Gamma; \Delta_1 \vdash t : \alpha \multimap \beta \quad \Gamma; \Delta_2 \vdash u : \alpha}{\Gamma; \Delta_1, \Delta_2 \vdash tu : \beta} \quad (\text{lin. app.})$$

NON LINEARITY

$$\Gamma; x : \alpha \vdash x : \alpha \quad (\text{lin. var.}) \qquad \Gamma, x : \alpha; \vdash x : \alpha \quad (\text{var.})$$

$$\frac{\Gamma; \Delta, x : \alpha \vdash t : \beta}{\Gamma; \Delta \vdash \lambda^\circ x. t : \alpha \multimap \beta} \quad (\text{lin. abs.})$$

$$\frac{\Gamma; \Delta_1 \vdash t : \alpha \multimap \beta \quad \Gamma; \Delta_2 \vdash u : \alpha}{\Gamma; \Delta_1, \Delta_2 \vdash tu : \beta} \quad (\text{lin. app.})$$

$$\frac{\Gamma, x : \alpha; \Delta \vdash t : \beta}{\Gamma; \Delta \vdash \lambda x. t : \alpha \rightarrow \beta} \quad (\text{abs.})$$

NON LINEARITY

$$\Gamma; x : \alpha \vdash x : \alpha \quad (\text{lin. var.}) \qquad \Gamma, x : \alpha; \vdash x : \alpha \quad (\text{var.})$$

$$\frac{\Gamma; \Delta, x : \alpha \vdash t : \beta}{\Gamma; \Delta \vdash \lambda^\circ x. t : \alpha \multimap \beta} \quad (\text{lin. abs.})$$

$$\frac{\Gamma; \Delta_1 \vdash t : \alpha \multimap \beta \quad \Gamma; \Delta_2 \vdash u : \alpha}{\Gamma; \Delta_1, \Delta_2 \vdash tu : \beta} \quad (\text{lin. app.})$$

$$\frac{\Gamma, x : \alpha; \Delta \vdash t : \beta}{\Gamma; \Delta \vdash \lambda x. t : \alpha \rightarrow \beta} \quad (\text{abs.})$$

$$\frac{\Gamma; \Delta \vdash t : \alpha \rightarrow \beta \quad \Gamma; \vdash u : \alpha}{\Gamma; \Delta \vdash tu : \beta} \quad (\text{app.})$$

DEPENDENT PRODUCTS

DEPENDENT PRODUCTS

TYPES DEPENDENT OF TERMS

DEPENDENT PRODUCTS

TYPES DEPENDENT OF TERMS

$gender = \{m \mid f\} : \text{type}$
 $number = \{s \mid p\} : \text{type}$
 $N : (gender, number) \text{ type}$
 $CHAT : N \ m \ s$
 $PETIT : (\Pi x : gender)(\Pi y : number) N \ x \ y \multimap N \ x \ y$