Expressing agent-opinions

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1 Introduction

Agent-opinions play a significant role in their various day-to-day social interactions. The concept of *opinion* is very subjective in nature, and in some sense, beyond truth or falsity. One can merely assert having a positive or a negative opinion about something or somebody, and verifying the truth of such opinion statements is not really meaningful.

In certain real-life situations, when one is faced with incomplete information, based on which decisions are needed to be taken, one often relies on her opinions about the world - with a *social* alternative being relying on other peoples' opinions also. Thus, intermingling of opinions often helps us to take decisive actions in presence of conflicting information.

The way we are considering "opinion" here is essentially close to a notion which is termed as "non-institutionalized public opinion" by Karl Popper [5]. As mentioned in [5], the non-institutionalized public opinion relates to the socially interactive existence of human beings, and as such is an interesting social concept to formalize. Popper further mentions the importance of investigating the logical character of such opinions of agents along with their interaction with other opinions. Opinion of an agent can be thought of as "unsupported belief" of an agent, that is, a belief which has not yet been provided with much rational support to make it true.

In general, besides our own opinions about the situation we are concerned with, we also have other agents' opinions. Moreover, all these opinions can influence each other resulting in some changes also. In this work, we recapitulate

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logical frameworks that allow us to represent such network of opinions, providing general rules for the way opinions interact and influence each other. In Section 2, we recall the logical language of [3]; it allows us to describe agent-opinions not only about facts but also about other agents. We present the semantic model in which such formulas are evaluated, providing a sound and complete axiom system for its validities. In [3], the authors do not consider properties relating *opinions about facts* with *opinions about agents*. Though in general imposing relations between them is an empirical task, there are situations in which the way this interplay happens becomes pretty obvious. We propose postulates covering the most relevant of such cases, together with proposing formulas of the language that characterize them. Then, in Section 3, we extend this logic of opinions to incorporate beliefs and preferences (also introduced in [3]), providing postulates relating the different concepts as well as their syntactic characterizations. Finally, we give pointers for further work in section 4.

2 A logic of opinions

The logic of opinions [3] basically represents situations comprising of agents and events, together with opinions of agents about these events and also about other agents. The logical framework underlying it is that of *Hybrid logic* ([6, 1]); it provides us with an uniform representation of the different types of nodes that we consider in the model. The syntax is defined in the following way.

Definition 2.1 (Language $\mathcal{LO}$)**.** *Let* $\texttt{PROP}$ *be a* finite *set of atomic propositions,* $\texttt{NOM}$ *be a set of world-names (nominals) and* $\texttt{AG}$ *be a* finite *set of agent-names. We assume that all these sets are pairwise disjoint. Formulas of the language of the* logic of opinions ($\mathcal{LO}$) *are given by*

$$\varphi ::= p \mid i \mid \neg\varphi \mid \varphi \vee \psi \mid \Box_a^+\varphi \mid \Box_a^-\varphi \mid \oplus_{a:b} \mid \ominus_{a:b} \mid @_i\varphi$$

where $p \in \texttt{PROP}$*,* $i \in \texttt{NOM}$ *and* $a, b \in \texttt{AG}$*. We assume a restricted language in the sense that nesting of opinion modalities are not allowed. In the formulas of the form* $\Box_a^+\varphi$ *and* $\Box_a^-\varphi$*, the formula* φ *is restricted to nominals, atomic propositions and their boolean and* $@_i$ *combinations.*

Formulas of the form $\Box_a^+\varphi$ and $\Box_a^-\varphi$ express agent a's opinion (positive and negative, respectively) about φ, and $\oplus_{a:b}$ and $\ominus_{a:b}$ express agent a's opinion (positive and negative, respectively) about agent b. Formulas of the form $@_i\varphi$ indicates that φ is the case in the world named by the nominal i. Other boolean connectives ($\wedge, \rightarrow, \leftrightarrow$) and the diamond versions of $\Box_a^+$ and $\Box_a^-$ are defined as usual.

As an example, the formula $\neg\Box_a^+\varphi \wedge \neg\Box_a^-\varphi$ is read as *"the agent does not have any opinion about* φ*"*, whereas $\Box_a^+\varphi \wedge \Box_a^-\varphi$ corresponds to *"a is undecided about* φ. In terms of epistemic attitudes of an agent, there is a difference between *having no opinion* and *being undecided* about a certain event. One can be undecided whether to take an umbrella or not while she is going out, but she may have no opinion about who should win the next Indian parliamentary elections, as she is simply not interested in the issue. On the other hand, in terms of opinions concerning other agents, these attitudes are typically indistinguishable.

Our semantic model is an extension of Kripke models. The main difference is that, besides nodes representing possible worlds, we also have nodes representing *agents*.

Definition 2.2 (Opinion models)**.** *Let PROP, NOM and AG be sets just as before. An* opinion model *is a graph-like structure* $M = \langle W, A, R^+, R^-, O^+, O^-, V, N\rangle$ *where the set of nodes is given by the union of the disjoint sets* W *and* A *(*world-nodes *and* agent-nodes*, respectively),* R^+ *and* R^- *denote binary relations from* A *to* W *(agents' positive and negative opinions about facts) and* O^+ *and* O^- *denote binary relations on* A *(agents' positive and negative opinions about agents). Finally,* $V : (\mathtt{PROP} \cup \mathtt{NOM}) \to \wp(W)$ *and* $N : \mathtt{AG} \to A$ *are functions, with* V *assigning a set of world-nodes to each atomic proposition and a singleton to each world-name, and* N *being an injection.*

We assume R^+ *and* R^- *to be* serial*: every agent has some positive as well as some negative opinion about certain facts. The opinion model* M *is* named *if every world-node in the model is the denotation of some nominal, that is, for each* $w \in W$*, there is a nominal* $i \in \mathtt{NOM}$*, such that* $V(i) = \{w\}$*.*

Agents' opinions are represented by relations between nodes in the model. Relations R^+ and R^- are used to define agents' positive and negative opinions about facts, and relations O^+ and (O^-n) are used to define agents' positive and negative opinions about agents. The use of nodes for representing agents allows us to represent the notion of opinion about agents, a concept whose definition is not clear in models where agents are characterized by the accessibility relations describing their knowledge, beliefs, preferences, etc.

Formulas in $\mathcal{LO}$ are interpreted in opinion models as follows.

Definition 2.3. *Given a named opinion model* $M = \langle W, A, R^+, R^-, O^+, O^-, V, N\rangle$ *and a world-node* $w \in W$*, the truth-value of formulas of* $\mathcal{LO}$ *in* M *at world* w *are given as follows:*

$(M,w) \vDash p$	*iff*	$w \in V(p)$
$(M,w) \vDash i$	*iff*	$\{w\} = V(i)$
$(M,w) \vDash \neg\varphi$	*iff*	$(M,w) \nvDash \varphi$
$(M,w) \vDash \varphi \vee \psi$	*iff*	$(M,w) \vDash \varphi$ *or* $(M,w) \vDash \psi$
$(M,w) \vDash \Box_a^+\varphi$	*iff*	*for all* $u \in W$ *s.t.* $R^+N(a)u$*, we have* $(M,u) \vDash \varphi$
$(M,w) \vDash \Box_a^-\varphi$	*iff*	*for all* $u \in W$ *s.t.* $R^-N(a)u$*, we have* $(M,u) \nvDash \varphi$
$(M,w) \vDash \oplus_{a:b}$	*iff*	$O^+N(a)N(b)$
$(M,w) \vDash \ominus_{a:b}$	*iff*	$O^-N(a)N(b)$
$(M,w) \vDash @_i\varphi$	*iff*	$(M,u) \vDash \varphi$*, where* $V(i) = \{u\}$

Note that, besides seriality for R^+ and R^-, we do not have any other restriction on the opinion relations. This gives us certain freedom with respect to the "consistency" of the opinions represented in the model. Though formulas like $\Box^+_a\varphi \wedge \Box^+_a\neg\varphi$ (*"agent a has a positive opinion about both φ and $\neg\varphi$"*) are indeed unsatisfiable, some others like $\Box^+_a\varphi \wedge \Box^-_a\varphi$ (*"agent a has both a positive and a negative opinion about φ"*) or $\oplus_{a:b} \wedge \ominus_{a:b}$ (*"agent a has both a positive and a negative opinion about agent b"*) do have a model.

There are two main reasons for this freedom. First, by using such a relaxed basic framework, we can go to particular "idealized" situations by simply imposing additional conditions that reflects intuitive ideas about how positive and negative opinions about agents and facts should be related (as we do in Section 2.2 and Section 3.1). Second, it allows us to look at these "idealizations" from a *dynamic* perspective, and consider them not as properties of every agents' opinion at every stage, but as the desirable outcome of a sequence of processes that revise and correct them. This will be one of our main focus for further work (cf. Section 4).

As mentioned earlier, our framework uses concepts of Hybrid Logics. We have names for every node in the model, though there are two different kinds of nodes: world-nodes and agent-nodes. Moreover, while world-nodes are standard possible worlds where formulas can be evaluated, the function of agent-nodes is to represent agents explicitly in the model (and not through her knowledge, beliefs, preferences, etc.), and therefore they do not behave as possible worlds.

A communication situation representing agents' opinion about events and about each other can be described by finite conjunction of modal formulas in $\mathcal{LO}$. Consider the following simple example:

> Suppose Professor Calculus wants to know how good a singer Bianca Castafiore is. In come Thomson and Thompson, and convey the following.
> - `Thomson`: "*She is a very good singer.*"
> - `Thompson`: "*Aha! I do not think so. I really dislike her singing*"

This network of opinions can be represented by the formula $\Box^+_a \wedge \Box^-_b p$, where a represents Thomson, b represents Thompson, and p expresses the fact that "Bianca Castafiore is a good singer". The obvious question here is what would Professor Calculus infer in this situation with conflicting opinions. We will come back to this example at the end of this section.

2.1 Axiom system

Using techniques of hybrid logic, we provide a sound and complete axiom system for $\mathcal{LO}$ (proved in [3]).

Theorem 1. *The validities of the logic $\mathcal{LO}$ in countable named opinion models are completely axiomatizable by the axioms and rules of the following table.*

$\vdash \varphi$ for φ a propositional tautology			
If $\vdash \varphi \to \psi$ and $\vdash \varphi$ then $\vdash \psi$			
$\vdash @_i(p \to q) \to (@_i p \to @_i q)$		$\vdash @_i j \leftrightarrow @_j i$	
$\vdash @_i p \leftrightarrow \neg @_i \neg p$		$\vdash @_i j \land @_j p \leftrightarrow @_i p$	
$\vdash i \land p \to @_i p$		$\vdash @_i @_j p \leftrightarrow @_j p$	
$\vdash @_i i$		If $\vdash \varphi$ then $\vdash @_i \varphi$	
$\vdash \Box_a^+(p \to q) \to (\Box_a^+ p \to \Box_a^+ q)$		$\vdash \Box_a^-(\neg p \land q) \to (\Box_a^- p \to \Box_a^- q)$	
$\vdash \Box_a^+ p \to \Diamond_a^+ p$	*(+ser)*	$\vdash \Box_a^- p \to \Diamond_a^- p$	*(-ser)*
$\vdash (\Diamond_a^+ i \land @_i p) \to \Diamond_a^+ p$	*(+translation)*	$\vdash (\Diamond_a^- i \land @_i \neg p) \to \Diamond_a^- p$	*(-translation)*
$\vdash \Diamond_a^+ @_i p \to @_i p$	*(+back)*	$\vdash \Diamond_a^- @_i p \to \neg @_i p$	*(-back)*
If $\vdash \varphi$ then $\vdash \Box_a^+ \varphi$	*(+gen)*	If $\vdash \neg\varphi$ then $\vdash \Box_a^- \varphi$	*(-gen)*
$\vdash \Diamond_a^+ \varphi \leftrightarrow @_i \Diamond_a^+ \varphi$	*(+@ agree)*	$\vdash \Diamond_a^- \varphi \leftrightarrow @_i \Diamond_a^- \varphi$	*(-@ agree)*
$\vdash \oplus_{a:b} \leftrightarrow @_i \oplus_{a:b}$	*($\oplus$ @ agree)*	$\vdash \ominus_{a:b} \leftrightarrow @_i \ominus_{a:b}$	*($\ominus$ @ agree)*
If $\vdash i \to \varphi$ then $\vdash \varphi$ for i not occurring in φ			
If $\vdash (@_i \Diamond_a^+ j \land @_j \varphi) \to \psi$ then $\vdash @_i \Diamond_a^+ (\varphi \to \psi)$ for $i \neq j$ and j not occurring in φ or ψ			
If $\vdash (@_i \Diamond_a^- j \land @_j \neg\varphi) \to \psi$ then $\vdash @_i \Diamond_a^- (\varphi \to \psi)$ for $i \neq j$ and j not occurring in φ or ψ			
If $\vdash \varphi$ then $\vdash \varphi\sigma$ where σ is a substitution that uniformly replaces atomic propositions by formulas, agent-names by agent-names and nominals by nominals.			

A detailed proof of the Theorem can be found in [3]; here we just describe the main idea.

Soundness can be proved easily. For completeness, we follow the idea of Chapter 7.3 of [2]. Any consistent set of $\mathcal{LO}$ formulas Σ can be extended to a *named* and *pasted* maximal consistent set Σ^+ from which we define the following sets for each $i \in \mathtt{NOM}$ and each $a \in \mathtt{AG}$):

$$\begin{aligned} \Delta_i &:= \{\varphi \mid @_i\varphi \in \Sigma^+\} \\ \Delta_a &:= \{\Diamond_a^+\varphi \mid \Diamond_a^+\varphi \in \Sigma^+\} \cup \{\Diamond_a^-\varphi \mid \Diamond_a^-\varphi \in \Sigma^+\} \cup \\ &\quad\ \{\oplus_{a:b} \mid \oplus_{a:b} \in \Sigma^+\} \cup \{\ominus_{a:b} \mid \oplus_{a:b} \in \Sigma^+\} \end{aligned}$$

Note that for every $a \in \mathtt{AG}$, Δ_a is never empty (because of *gen* and *ser*); hence, different agent names generate different Δ sets.

From Σ^+ we build a canonical opinion model where world nodes are Δ_i sets and agent-nodes are Δ_a sets. The canonical relations are defined in the following way: for opinions about facts we have $R^+\Delta_a\Delta_i$ ($R^-\Delta_a\Delta_i$) *iff* for all formulas φ, $\varphi \in \Delta_i$ implies $\Diamond_a^+\varphi \in \Delta_a$ ($\neg\varphi \in \Delta_i$ implies $\Diamond_a^-\varphi \in \Delta_a$); for opinions about agents we have $O^+\Delta_a\Delta_b$ ($O^-\Delta_a\Delta_b$) *iff* $\oplus_{a:b} \in \Delta_a$ ($\ominus_{a:b} \in \Delta_a$). The functions V and N are given by $\Delta_i \in V(x)$ *iff* $x \in \Delta_i$ (for $x \in \mathtt{PROP} \cup \mathtt{AG}$) and by $N(a) := \Delta_a$. It can be verified that the constructed structure is actually an opinion model (the seriality of R^+ and R^- uses the *ser*, *agree* and *translation* axioms plus the fact that Σ^+ is *pasted*). The proof of the *existence* lemma is standard (follows from that of the hybrid logic) just as the proofs of the usual cases of the *truth* lemma. For the rest, that is for the cases of opinion formulas, the proof follows from the *existence* lemma and the *agree* axioms.

2.2 Relation between opinions

The just defined system allows us to talk about agents' opinions in a very general sense. The opinion relations do not interact with each other and as such is not very interesting. Relating opinions about agents with opinions about facts is not so simple: what should agent a do if she wants to form an opinion about some fact φ, and while having a positive opinion about both agents b and c, finds out that b and c disagree about φ? Nevertheless, there are some cases where it is clear what should be the case: for example, if two agents share the same opinions about all the relevant facts, then it is appropriate for them to have a positive opinion about each other.

Here we present a set of *postulates* stating how opinions should be related in special cases, providing also formulas of $\mathcal{LO}$ that characterize them. Note that, given our semantic definition, positive opinion about facts are closed under following well-known rule: if an agent has a positive opinion about both φ and $\varphi \to \psi$, then she also has a positive opinion about ψ. Negatives facts behave in a somewhat similar fashion: if an agent has a negative opinion about both φ and $\neg\varphi \wedge \psi$, then she also has a negative opinion about ψ. Our postulates just deal with propositional letters, but because of the rules above they can be extended to complex formulas.

Let a, b be agent-names and p be an atomic proposition. The first two postulates express an ideal situation in which opinions about agents influence opinions about facts, while the last two describe an ideal relation in which opinions about facts influence opinions about agents.

P1 If every agent about which a has a positive opinion has a positive opinion about p and every agent about which a has a negative opinion has a negative opinion about p, then a has a positive opinion about p.

P2 If every agent about which a has a positive opinion has a negative opinion about p and every agent about which a has a negative opinion has a positive opinion about p, a has a negative opinion about p.

P3 If agents a and b have similar opinions about each atomic proposition p (on which they both have some opinion), then agent a has a positive opinion about agent b.

P4 If agents a and b have different opinions about each atomic proposition p (on which they both have some opinion), then agent a has a negative opinion about agent b.

In order to provide formulas characterizing the postulates, the following definitions will be useful. Given a model M and an agent-name $a \in \mathtt{AG}$, define the set of agents about which a has a *positive* opinion and the set of agents about which a has a *negative* opinion as follows.

$$\mathrm{Pos}_{\mathtt{AG}}(a) \coloneqq \{b \in \mathtt{AG} \mid O^{+}N(a)N(b)\} \qquad \mathrm{Neg}_{\mathtt{AG}}(a) \coloneqq \{b \in \mathtt{AG} \mid O^{-}N(a)N(b)\}$$

Similarly, for an agent-name $a \in \mathtt{AG}$, we define the set of atomic propositions about which a has a *positive* opinion and the set of atomic propositions about which a has a *negative* opinion as follows.

$$\mathrm{Pos}_{\mathtt{PROP}}(a) \coloneqq \{p \in \mathtt{PROP} \mid R^{+}N(a)u \text{ implies } u \in V(p)\}$$
$$\mathrm{Neg}_{\mathtt{PROP}}(a) \coloneqq \{p \in \mathtt{PROP} \mid R^{-}N(a)u \text{ implies } u \notin V(p)\}$$

The postulates we have defined can be expressed by the following formulas of $\mathcal{LO}$:

$$\mathbf{A1}: \Big(\bigwedge_{b \in \mathrm{Pos}_{\mathtt{AG}}(a)} \Box_b^{+} p \wedge \bigwedge_{b \in \mathrm{Neg}_{\mathtt{AG}}(a)} \Box_b^{-} p \Big) \to \Box_a^{+} p$$
$$\mathbf{A2}: \Big(\bigwedge_{b \in \mathrm{Pos}_{\mathtt{AG}}(a)} \Box_b^{-} p \wedge \bigwedge_{b \in \mathrm{Neg}_{\mathtt{AG}}(a)} \Box_b^{+} p \Big) \to \Box_a^{-} p$$
$$\mathbf{A3}: \Big(\bigwedge_{p \in \mathrm{Pos}_{\mathtt{PROP}}(a)} \Box_b^{+} p \wedge \bigwedge_{p \in \mathrm{Neg}_{\mathtt{PROP}}(a)} \Box_b^{-} p \Big) \to \oplus_{a:b}$$
$$\mathbf{A4}: \Big(\bigwedge_{p \in \mathrm{Pos}_{\mathtt{PROP}}(a)} \Box_b^{-} p \wedge \bigwedge_{p \in \mathrm{Neg}_{\mathtt{PROP}}(a)} \Box_b^{+} p \Big) \to \ominus_{a:b}$$

Not only that: the given formulas characterize the corresponding properties on opinion models.

Theorem 2. *The validities of the logic $\mathcal{LO}$ together with the postulates **P1-P4** are completely axiomatizable by axioms and rules shown in Theorem 1 together with axioms **A1-A4**.*

Let us now analyze the example mentioned at the beginning of this section. If Professor Calculus (c) has a positive opinion about Thomson and a negative opinion about Thompson, the whole situation can be represented as,

$$\Box_a^{+} p \wedge \Box_b^{-} p \wedge \oplus_{c:a} \wedge \ominus_{c:b}$$

From **A1**, it easily follows that $\Box_c^{+} p$.

3 Opinions, beliefs and preferences

When a complex multi-agent system such as a group of humans get involved into certain activities, viz. interacting with each other, taking some joint decisions or even playing group games, various kinds of epistemic attitudes come into play. We have provided a meaningful logical model of one of these attitudes, viz. opinion; here we will talk about the inter-relationship between opinions, beliefs and preferences.

Huge amount of debates and discussions can be found in the philosophical literature regarding the distinction between *opinions* and *beliefs*. For example, one can have a look at [4], where *opinions* are basically considered as *judgements* satisfying certain conditions (according to Kant).

By looking closer to the semantic definition for opinion formulas, we can notice that, just like the satisfaction formulas $@_i \varphi$ in hybrid logic, the truth-value of the opinion formulas do not depend on the evaluation point w. In particular, further nesting of formulas expressing opinions of agents about facts, if allowed, would not affect their truth-value. Moreover nesting of opinions is not very meaningful also, whereas, when we talk about *beliefs*, the nesting phenomenon occurs naturally.

In [3], $\mathcal{LO}$ is extended by adding the notions of *beliefs about facts* and *preferences about agents* so as to form the logic $\mathcal{LOBP}$, whose syntax and semantics are given in the following. Its main drawback is the fact that the interplay between opinions, beliefs and preferences is yet to be considered. As in the previous section, we propose some postulates to govern this interplay.

Definition 3.1 (Language $\mathcal{LOBP}$)**.** *Let* `PROP` *be a set of atomic propositions,* `NOM` *be a set of world-names and* `AG` *be a set of agent-names. We suppose that all these sets are pairwise disjoint. Formulas of the language of* logic of opinions, beliefs and preferences ($\mathcal{LOBP}$) *are given by*

$$\varphi ::= p \mid i \mid \neg\varphi \mid \varphi \vee \psi \mid \Box_a^+\varphi \mid \Box_a^-\varphi \mid \oplus_{a:b} \mid \ominus_{a:b} \mid @_i\varphi \mid B_a\varphi \mid a \preceq_c b$$

where $p \in \mathtt{PROP}$*,* $i \in \mathtt{NOM}$ *and* $a, b, c \in \mathtt{AG}$*.*

The intuitive reading of $B_a\varphi$ is that 'a believes in φ', and that of $a \preceq_c b$ is 'c prefers b to a'.

Definition 3.2 (Opinion, belief and preference (OBP) models)**.** *Let* `PROP` *be a set of atomic propositions,* `NOM` *be a set of world-names and* `AG` *be a set of agent-names. An* opinion, belief and preference model *is a graph-like structure*

$$M = \langle W, A, R^+, R^-, O^+, O^-, \{R_a \mid a \in \mathtt{AG}\}, \{\leq_a \mid a \in \mathtt{AG}\}, V, N\rangle$$

where $\langle W, A, R^+, R^-, O^+, O^-, V, N\rangle$ *is an opinion model as before, extended with a* serial, transitive and euclidean *relation* R_a *and a* reflexive and transitive *relation* $\leq_a$ *for each agent-name* a*. The properties of* R_a *are the standard ones when dealing with beliefs, and those of* $\leq_a$ *make it a preorder.*

Definition 3.3. *Given an opinion, belief and preference model* M *and a world-node* $w \in W$*, the truth-value of formulas of* $\mathcal{LOBP}$ *already in* $\mathcal{LO}$ *is given as before. For the new formulas, we have the following:*

$(M, w) \vDash B_a\varphi$	*iff*	*for all* $u \in W$ *s.t.* $R_a wu$*, we have* $(M, u) \vDash \varphi$
$(M, w) \vDash a \preceq_c b$	*iff*	$N(a) \leq_c N(b)$

Validities of $\mathcal{LOBP}$ in named OBP-models are axiomatizable by the system of Theorem 1 together with standard axioms expressing the properties of the new relations with the usual additions for nominals [3].

Theorem 3. *The validities of the logic* $\mathcal{LOBP}$ *in countable named OBP-models are completely axiomatizable by the axioms and rules of the following table.*

LO axioms and rules	
Belief axioms:	Preference axioms:
$\vdash B_a(p \to q) \to (B_a p \to B_a q)$	$\vdash b \preceq_a b$
$\vdash \langle B_a\rangle @_i p \to @_i p$	$\vdash (b \preceq_a c) \wedge (c \preceq_a d) \to (b \preceq_a d)$
$\vdash B_a i \to \langle B_a\rangle i p$	$\vdash (b \preceq_a c) \vee (c \preceq_a b)$
$\vdash B_a i \to B_a B_a i$	$\vdash (b \preceq_a c) \leftrightarrow @_i(b \preceq_a c)$
$\vdash \neg B_a i \to B_a \neg B_a i$	
If $\vdash \varphi$ then $\vdash B_a\psi$	
If $\vdash (@_i\langle B_a\rangle j \wedge @_j\varphi) \to \psi$ then $\vdash @_i\langle B_a\rangle\varphi \to \psi$	
for $i \neq j$ and j not occurring in φ or ψ	

3.1 Postulates

We now list some intuitive interactive properties in this OBP-model, which are basically self-explanatory. The corresponding expressions in the language of $\mathcal{LOBP}$ are also provided.

P5 If a has positive opinion about b, and negative opinion about c, then a prefers b to c.

$(\oplus_{a:b} \wedge \ominus_{a:c}) \to c \preceq_a b$ (**A5**)

P6 If a prefers b to c, and b thinks positively about φ, whereas c thinks negatively about the same, then a may have a positive opinion about φ.

$(c \preceq_a b \wedge \Box^+_b \varphi \wedge \Box^-_c \varphi) \to \Diamond^+_a \varphi$ (**A6**)

P7 If a prefers b to c, and b thinks negatively about φ, whereas c thinks positively about the same, then a may have a negative opinion about φ.

$(c \preceq_a b \wedge \Box^+_b \varphi \wedge \Box^-_c \varphi) \to \Diamond^+_a \varphi$ (**A7**)

Theorem 4. *The validities of the logic $\mathcal{LOBP}$ together with the postulates **P1-P7** are completely axiomatizable by axioms and rules shown in Theorem 3 together with axioms **A1-A7**.*

4 Further work

One immediate step further in the present scenario is to consider the dynamical aspects of these concepts: how opinions, beliefs and preferences change through diverse information process? Among such dynamic extensions, there is a significant issue relevant for the postulates. We have presented them as timeless properties: if the antecedent is true, then the consequent should be true *at the same stage*. But we can look at them *dynamically* as well: if the antecedent is true, then the consequent should be true *at some later stage*. This interpretation, closer to the step-by-step reasoning of real agents, raises several interesting questions about the definition of such dynamic operations and the conditions under which this process leads to a stable outcome.

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