Reasoning about collectively accepted group beliefs

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Abstract

A proof-theoretical treatment of collectively accepted group beliefs is presented through a multi-agent sequent system for an axiomatization of the logic of acceptance. The system is based on a labelled sequent calculus for propositional multi-agent epistemic logic with labels that correspond to possible worlds and a notation for internalized accessibility relations between worlds. The system is shown to be contraction- and cut-free. Extensions of the basic system are also considered, in particular with rules that allow the possibility of operative members or legislators.

1 Introduction

The study of collective attitudes has been in the focus of the philosophical literature concerned with *collective intentionality* [9, 21, 25]. One outcome of this area of research has been an understanding of the nature of collectively accepted group beliefs and their importance in creating the social environment. Attempts have been made recently to formalize reasoning about such collective attitudes. One motivation comes from theoretical social sciences, especially theories of social choice that study the aggregation of individual attitudes, especially preferences and judgements into collective attitudes. Formal systems of logic have been used to gain a more precise understanding of the properties of these aggregation processes, see e.g. [20, 26]. Another motivation comes from areas of application such as distributed artificial intelligence that aims at constructing multi-agent systems in which the agents can reason about the attitudes of other agents [22]. Various multi-agent logics have been presented to this task. Most of them are multi-modal logics that extend traditional modal logics, in particular epistemic logic.

The focus has been until recently on individual attitudes and what are known as *summative* collective attitudes, which can be defined in terms of individual attitudes, in particular, shared beliefs, mutual beliefs, distributed knowledge, and common knowledge [2]. In recent work, also *non-summative* collective attitudes, such as *group beliefs*, have received attention [4, 3, 7, 5, 13]. Group beliefs are taken to be collectively intentional attitudes that are based on what the group members accept as the group's belief [8, 24]. Thus, group beliefs do not reduce to individual beliefs but are properly attributed only to the collectivity. The fact that all of the group members believe that A is neither sufficient nor necessary for a group belief that A. It is required for group belief that the group

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members take A to be true when they are acting in the group context, that is, that the individuals accept A when they are acting as group members. The distinction between belief and acceptance allows reasoning about individual and collective attitudes in their proper context without attributing contradictory beliefs to the agents. The concept of acceptance allows inferences about public commitments of agents, because from their communication only their acceptances can be inferred, not necessarily their beliefs (see e.g. [6] for discussion).

In this paper, we present a sequent calculus system that allows to make proofs about collective attitudes. We take a formalization of this kind as crucial for the implementation of reasoning about collective attitudes. A closely related approach (that we found after having developed our own) is presented in [1] in which a tableau system for the logic is presented. We employ the general method for constructing modal sequent calculi presented in [15] (an introduction to sequent calculus and more generally to structural proof theory is found in [18]). The approach followed here is similar to the one in [11] in which the modal operator $\Box$ is replaced by knowledge operators $\mathcal{K}_a$ for individual agents $a \in G$ and an operator for distributed knowledge among agents in a group. In this paper, our focus is on group belief that we take to amount to a collective acceptance of a proposition by the group members to represent a view of the group [8, 24, 10]. Of the recent attempts to formalize such non-summative group beliefs (see [4, 3, 7, 5, 13]), we have here selected [13], which is formally sophisticated and quite faithful to philosophical accounts of group beliefs. The methods presented could be adapted for the other logics with minor modifications.

2 Background on labelled sequent systems

To maintain the presentation self-contained, we briefly recall in this section the background of our method, presented in [17, 18, 14], for the development of cut-free labelled systems for multi-modal logics.

For extensions of classical predicate logic, the starting point is the contraction- and cut-free sequent calculus **G3c** (cf. [18, 23] for the rules). We recall that all the rules of **G3c** are invertible and all the structural rules are admissible. Weakening and contraction are in addition *height-preserving-* (*hp-*) admissible, that is, whenever their premisses are derivable, so also is their conclusion, with at most the same derivation height (the *height* of a derivation is its height as a tree, that is, the length of its longest branch). Moreover, the calculus enjoys hp-admissibility of substitution. Invertibility of the rules of **G3c** is also height-preserving (*hp-invertible*). Detailed proofs can be found in chapters 3 and 4 of [18].

These remarkable structural properties of **G3c** are maintained in extensions of the logical calculus with suitably formulated rules that represent axioms for specific theories. Universal axioms are first transformed, through the rules of **G3c**, into a normal form that consists of conjunctions of formulas of the form $P_1 \& \ldots \& P_m \supset Q_1 \vee \ldots \vee Q_n$, where all P_i, Q_j are atomic; then implication reduces to the succedent if $m = 0$, and the latter is $\bot$ if $n = 0$. The universal closure of any such formula is called a *regular* formula. We abbreviate the multiset $P_1, \ldots, P_m$ as $\overline{P}$. Each conjunct is then converted into a schematic rule, called the *regular rule scheme*, of the form

$$\frac{Q_1, \overline{P}, \Gamma \Rightarrow \Delta \quad \ldots \quad Q_n, \overline{P}, \Gamma \Rightarrow \Delta}{\overline{P}, \Gamma \Rightarrow \Delta}\, Reg$$

By this method, all universal theories can be formulated as contraction- and cut-free systems of sequent calculi.

In [14], the method is extended to cover also *geometric theories*, that is, theories axiomatized by geometric implications. We recall that a *geometric formula* is a formula that does not contain $\supset$, $\neg$, or $\forall$, and a *geometric implication* is a sentence of the form $\forall\overline{z}(A \supset B)$ where A and B are geometric formulas. Geometric implications can be reduced to a normal form that consists of conjunctions of formulas, called *geometric axioms*, of the form

$$\forall\overline{z}(P_1 \& \ldots \& P_m \supset \exists\overline{x}(M_1 \vee \ldots \vee M_n))$$

where each M_j is a conjunction of atomic formulas, $Q_{j_1}, \ldots, Q_{j_{k_p}}$. For simplicity, we assume that the sequence $\overline{x}$ of bound variables has length 1. Without loss of generality, no x_i is free in any P_j. Note that regular formulas are degenerate cases of geometric implications, with neither conjunctions nor existential quantifications to the right of the implication. The *geometric rule scheme* for geometric axioms takes the form

$$\frac{\overline{Q}_1(y_1/x_1), \overline{P}, \Gamma \Rightarrow \Delta \quad \ldots \quad \overline{Q}_n(y_n/x_n), \overline{P}, \Gamma \Rightarrow \Delta}{\overline{P}, \Gamma \Rightarrow \Delta}\ {\scriptstyle GRS}$$

where $\overline{Q}_j$ and $\overline{P}$ indicate the multisets of atomic formulas $Q_{j_1}, \ldots Q_{j_{k_j}}$ and $P_1, \ldots, P_m$, respectively, and the eigenvariables $y_1, \ldots, y_n$ of the premisses are not free in the conclusion. We use the notation $A(y/x)$ to indicate A after the substitution of the term y for the variable x.

In order to maintain admissibility of contraction in the extensions with regular and geometric rules, the formulas $P_1, \ldots, P_m$ in the antecedent of the conclusion of the scheme have, as indicated, to be repeated in the antecedent of each of the premisses. In addition, whenever an instantiation of free parameters in atoms produces a duplication (two identical atoms) in the conclusion of a rule instance, say $P_1, \ldots, P, P, \ldots, P_m, \Gamma \Rightarrow \Delta$, there is of course a corresponding duplication in each premiss. The *closure condition* imposes the requirement that the rule with the duplication P, P contracted into a single P, both in the premisses and in the conclusion, be added to the system of rules. For each axiom system, there is only a bounded number of possible cases of contracted rules to be added, very often none at all, so the condition is unproblematic.

The main result for such extensions is the following (Theorems 4 and 5 from [14]):

Theorem 1. *The structural rules of* Weakening*,* Contraction *and* Cut *are admissible in all extensions of* **G3c** *with the geometric rule-scheme and satisfying the closure condition.* Weakening *and* Contraction *are hp-admissible.*

The method of extension of sequent calculi can be applied not only to the proof theory of specific theories such as lattice theory, arithmetic, and geometry [19], but also to the proof theory of non-classical logics. In [15], rules expressing properties of binary relations are added to a basic labelled sequent calculus for the normal modal logic **K** in such a way that complete systems for all the modal logics characterized by geometric frame conditions are obtained. The basic labelled sequent calculus is obtained by prefixing with labels the formulas in the rules of the sequent calculus for the propositional part of **G3c**. As initial sequents we take any of the form $x : P, \Gamma \Rightarrow \Delta, x : P$ for atomic P. In each rule, the active and principal formulas are prefixed by the same label. This corresponds to the classical explanation of truth in Kripke semantics, flat on all the propositional logical constants.

For instance, the rules for conjunction are

$$\frac{x : A, x : B, \Gamma \Rightarrow \Delta}{x : A \& B, \Gamma \Rightarrow \Delta}\,{\scriptstyle L\&} \qquad \frac{\Gamma \Rightarrow \Delta, x : A \quad \Gamma \Rightarrow \Delta, x : B}{\Gamma \Rightarrow \Delta, x : A \& B}\,{\scriptstyle R\&}$$

and those for implication are

$$\frac{\Gamma \Rightarrow \Delta, x : A \quad x : B, \Gamma \Rightarrow \Delta}{x : A \supset B, \Gamma \Rightarrow \Delta}\,{\scriptstyle L\supset} \qquad \frac{x : A, \Gamma \Rightarrow \Delta, x : B}{\Gamma \Rightarrow \Delta, x : A \supset B}\,{\scriptstyle R\supset}$$

The rules for the modal operator $\Box$ are obtained similarly from its semantical explanation in terms of possible worlds

$$x : \Box A \textit{ iff for all } x,\ xRy \textit{ implies } y : A$$

that gives the rules

$$\frac{y : A, x : \Box A, xRy, \Gamma \Rightarrow \Delta}{x : \Box A, xRy, \Gamma \Rightarrow \Delta}\,{\scriptstyle L\Box} \qquad \frac{xRy, \Gamma \Rightarrow \Delta, y : A}{\Gamma \Rightarrow \Delta, x : \Box A}\,{\scriptstyle R\Box}$$

with the *"variable condition"* in $R\Box$ that y is *fresh*, i.e. not free in the conclusion.

The resulting sequent calculus, called **G3K**, gives a complete system for the basic normal modal logic **K**. This logic is characterized by arbitrary frames; correspondingly, there are no rules for the accessibility relation. The sequent calculi for extensions of **K** such as the modal logics **T, K4, KB, S4, B, S5** are obtained by adding to **G3K** the rules that express their *frame conditions*, i.e., the properties of the accessibility relation that characterize their frames. For instance, a sequent calculus for the modal logic **S4** is obtained by adding the rules for reflexivity and transitivity of the accessibility relation

$$\frac{xRx, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta}\,{\scriptstyle Refl} \qquad \frac{xRz, xRy, yRz, \Gamma \Rightarrow \Delta}{xRy, yRz, \Gamma \Rightarrow \Delta}\,{\scriptstyle Trans}$$

We recall from [15] the following properties of any extension **G3K*** of **G3K** with geometric rules for the frame conditions:

Theorem 2. *1. All sequents of the form* $x : A, \Gamma \Rightarrow \Delta, x : A$ *are derivable in* **G3K***.
2. All sequents of the form $\Rightarrow x : \Box(A \supset B) \supset (\Box A \supset \Box B)$ *are derivable in* **G3K***.
3. The substitution *rule*

$$\frac{\Gamma \Rightarrow \Delta}{\Gamma(y/x) \Rightarrow \Delta(y/x)}\,{\scriptstyle (y/x)}$$

is hp-admissible in **G3K***.
4. The rules of Weakening

$$\frac{\Gamma \Rightarrow \Delta}{x : A, \Gamma \Rightarrow \Delta}\,{\scriptstyle LW} \qquad \frac{\Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, x : A}\,{\scriptstyle RW} \qquad \frac{\Gamma \Rightarrow \Delta}{xRy, \Gamma \Rightarrow \Delta}\,{\scriptstyle LW_R}$$

are hp-admissible in **G3K***.
5. The Necessitation *rule*

$$\frac{\Rightarrow x : A}{\Rightarrow x : \Box A}\,{\scriptstyle Nec}$$

is admissible in **G3K***.
6. For each frame condition, the corresponding modal axiom is derivable in **G3K***.

7. *All the primitive rules of* **G3K*** *are hp-invertible.*

8. *The rules of* Contraction

$$\frac{x:A, x:A, \Gamma \Rightarrow \Delta}{x:A, \Gamma \Rightarrow \Delta}{\scriptstyle L\text{-}Ctr} \qquad \frac{xRy, xRy, \Gamma \Rightarrow \Delta}{xRy, \Gamma \Rightarrow \Delta}{\scriptstyle L\text{-}Ctr_R} \qquad \frac{\Gamma \Rightarrow \Delta, x:A, x:A}{\Gamma \Rightarrow \Delta, x:A}{\scriptstyle R\text{-}Ctr}$$

are hp-admissible in **G3K***.

9. *The* Cut *rule*

$$\frac{\Gamma \Rightarrow \Delta, x:A \quad x:A, \Gamma' \Rightarrow \Delta'}{\Gamma, \Gamma' \Rightarrow \Delta, \Delta'}{\scriptstyle Cut}$$

is admissible in **G3K***.

In multi-modal logics, there is not only one but many accessibility relations, each defining a corresponding modal operator. In multi-agent epistemic logics, the accessibility relations are indexed over a set of agents, and the modality defined by each of these is an individual's knowledge operator. The intersection of the accessibility relations gives then the accessibility relation for the modality of distributed knowledge. The results in [11] exemplify the backbone of the method for multimodal logics: First we give the rules for the accessibility relations, including the rules for obtaining other accessibility relations from given ones, in the form of rules that follow the regular or the geometric rule scheme. Then we obtain the rules for the corresponding modalities from their explanation in terms of Kripke semantics. Once the structural properties are established, completeness with respect to a Hilbert-style axiomatization follows from the derivability of the characteristic axioms in the system.

3 The system G3KA

We shall follow the axiomatization for the logic of acceptance given in [13] but use a slightly different notation. We denote the collective acceptance of A by $\mathcal{A}_{g:i}A$ meaning that the group g, in context i, believes that A. This is to be interpreted as the agents in g having accepted that A is the view of their group in the context of an institution i. A standard possible worlds semantics is considered, with W a non-empty set of possible worlds and $R_{g:i}$ the accessibility relation that corresponds to the modality $\mathcal{A}_{g:i}$.

As our basic system we use the propositional part of the system **G3c** given in [18] and extend it with the rules for modalities and acceptance relations as explained in the previous section. In complete analogy to the rules for $\Box$, we define the rules for the acceptance modality starting from their explanation in terms of relational semantics:

$$x \Vdash \mathcal{A}_{g:i}A \ \text{ iff } \ \forall y(xR_{g:i}y \rightarrow y \Vdash A)$$

The rules we obtain are the following:

$$\frac{xR_{g:i}y, \Gamma \Rightarrow \Delta, y:A}{\Gamma \Rightarrow \Delta, x:\mathcal{A}_{g:i}A}{\scriptstyle R\mathcal{A}_{g:i}} \qquad \frac{y:A, y:\mathcal{A}_{g:i}A, xR_{g:i}y, \Gamma \Rightarrow \Delta}{x:\mathcal{A}_{g:i}A, xR_{g:i}y, \Gamma \Rightarrow \Delta}{\scriptstyle L\mathcal{A}_{g:i}}$$

Rule $R\mathcal{A}_{g:i}$ has the variable condition that y must not appear in the conclusion.

In [13], the following semantic constraints are imposed on the frames, where $R_{h:j}(x)$ denotes the set $\{z \in W \mid xR_{h:j}z\}$:

S.1 If $h \subseteq g$ and $y \in R_{h:j}(x)$, then $R_{g:i}(y) \subseteq R_{g:i}(x)$

S.2 If $h \subseteq g$ and $y \in R_{h:j}(x)$, then $R_{g:i}(x) \subseteq R_{g:i}(y)$

S.3 If $h \subseteq g$ and $R_{g:i}(x) \neq \emptyset$, then $R_{h:i}(x) \subseteq R_{g:i}(x)$

S.4 If $y \in R_{g:i}(x)$, then $y \in \bigcup_{k \in g} R_{k:i}(y)$

S.5 If $h \subseteq g$ and $R_{g:i}(x) \neq \emptyset$, then $R_{h:i}(x) \neq \emptyset$

Once the set-theoretic definitions have been unfolded, these constraints are converted into syntactic rules after the pattern of the regular rule scheme or of the geometric rule scheme recalled in the previous section:

$$\frac{xR_{g:i}z, h \subseteq g, xR_{h:j}y, yR_{g:i}z, \Gamma \Rightarrow \Delta}{h \subseteq g, xR_{h:j}y, yR_{g:i}z, \Gamma \Rightarrow \Delta} \; RS.1$$

$$\frac{yR_{g:i}z, h \subseteq g, xR_{h:j}y, xR_{g:i}z, \Gamma \Rightarrow \Delta}{h \subseteq g, xR_{h:j}y, xR_{g:i}z, \Gamma \Rightarrow \Delta} \; RS.2$$

$$\frac{xR_{g:i}z, h \subseteq g, xR_{g:i}y, xR_{h:i}z, \Gamma \Rightarrow \Delta}{h \subseteq g, xR_{g:i}y, xR_{h:i}z, \Gamma \Rightarrow \Delta} \; RS.3$$

$$\frac{\{yR_{k:i}y, xR_{g:i}y, \Gamma \Rightarrow \Delta\}_{k \in g}}{xR_{g:i}y, \Gamma \Rightarrow \Delta} \; RS.4$$

$$\frac{xR_{h:i}z, h \subseteq g, xR_{g:i}y, \Gamma \Rightarrow \Delta}{h \subseteq g, xR_{g:i}y, \Gamma \Rightarrow \Delta} \; RS.5$$

Rule *RS.4* has a finite number of premisses, one for each element of the group g[1], and *RS.5* has the condition that z must not occur in the conclusion. We call the resulting system **G3KA**.

Lorini *et al.* [13] present an axiomatization of the logic of acceptance. The inference rules are the standard ones, *modus ponens* and *necessitation*, and the axioms, in addition to the standard ones (propositional tautologies and the axiom of K) are as follows:

PAccess $\mathcal{A}_{g:i}A \supset \mathcal{A}_{h:j}\mathcal{A}_{g:i}A$ if $h \subseteq g$.

NAccess $\sim\mathcal{A}_{g:i}A \supset \mathcal{A}_{h:j} \sim\mathcal{A}_{g:i}A$ if $h \subseteq g$.

Inc $(\sim\mathcal{A}_{g:i}\bot \wedge \mathcal{A}_{g:i}A) \supset \mathcal{A}_{h:i}A$ if $h \subseteq g$.

Unanim $\mathcal{A}_{g:i}(\bigwedge_{k \in g} \mathcal{A}_{k:i}A \supset A)$

Mon $\sim\mathcal{A}_{g:i}\bot \supset \sim\mathcal{A}_{h:i}\bot$ if $h \subseteq g$.

The following lemma, used for proposition 4 below, proves in our system the empirical social fact that disagreement persists in the enlargement of a group, unless additional assumptions such as the presence of authoritative members are added:

Lemma 3. *The sequent* $h \subseteq g, x : \mathcal{A}_{h:i}\bot \Rightarrow x : \mathcal{A}_{g:i}\bot$ *is derivable in* **G3KA**.

[1] By using the geometric rule scheme with an eigenvariable ranging over elements of g, the rule can be generalized to the case in which the group is not given as a finite list.

Proof. We have the following derivation

$$\dfrac{\dfrac{\dfrac{xR_{h:i}z, h \subseteq g, xR_{g:i}y, z:\bot, x:\mathcal{A}_{h:i}\bot \Rightarrow y:\bot}{xR_{h:i}z, h \subseteq g, xR_{g:i}y, x:\mathcal{A}_{h:i}\bot \Rightarrow y:\bot}\,{\scriptstyle L\mathcal{A}_{h:i}}}{h \subseteq g, xR_{g:i}y, x:\mathcal{A}_{h:i}\bot \Rightarrow y:\bot}\,{\scriptstyle RS.5}}{h \subseteq g, x:\mathcal{A}_{h:i}\bot \Rightarrow x:\mathcal{A}_{g:i}\bot}\,{\scriptstyle R\mathcal{A}_{g:i}}$$

where the topsequent is an instance of $L\bot$. □

Observe that the sequent that expresses persistence of agreement, obtained by replacing $\bot$ with an arbitrary formula A, is instead not derivable. This is seen by inspection of the small set of possible applicable rules at each step of the root-first proof search.

Proposition 4. *The axioms* PAccess*,* NAccess*,* Inc*,* Unanim*, and* Mon *are derivable in* **G3KA**.

Proof. Axiom *PAccess* can be derived in a root-first fashion, using the corresponding rule *RS.1*, as follows:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{z:A, xR_{g:i}z, h \subseteq g, xR_{h:j}y, yR_{g:i}z, x:\mathcal{A}_{g:i}A \Rightarrow z:A}{xR_{g:i}z, h \subseteq g, xR_{h:j}y, yR_{g:i}z, x:\mathcal{A}_{g:i}A \Rightarrow z:A}\,{\scriptstyle L\mathcal{A}_{g:i}}}{h \subseteq g, xR_{h:j}y, yR_{g:i}z, x:\mathcal{A}_{g:i}A \Rightarrow z:A}\,{\scriptstyle RS.1}}{h \subseteq g, xR_{h:j}y, x:\mathcal{A}_{g:i}A \Rightarrow y:\mathcal{A}_{g:i}A}\,{\scriptstyle R\mathcal{A}_{g:i}}}{h \subseteq g, x:\mathcal{A}_{g:i}A \Rightarrow x:\mathcal{A}_{h:j}\mathcal{A}_{g:i}A}\,{\scriptstyle R\mathcal{A}_{h:j}}}{h \subseteq g \Rightarrow x:\mathcal{A}_{g:i}A \supset \mathcal{A}_{h:j}\mathcal{A}_{g:i}A}\,{\scriptstyle R\supset}$$

The uppermost sequent is clearly derivable because it contains the same formula on both sides of the sequent arrow.

The derivation of axiom *NAccess* by rule *RS.2* is similar.

Axiom *Inc* can be derived using the corresponding rule *RS.3*, as follows:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{z:A\ldots \Rightarrow y:\bot, z:A}{xR_{g:i}z, xR_{h:i}z, xR_{g:i}y, x:\mathcal{A}_{g:i}A, h \subseteq g \Rightarrow y:\bot, z:A}\,{\scriptstyle L\mathcal{A}_{g:i}}}{xR_{h:i}z, xR_{g:i}y, x:\mathcal{A}_{g:i}A, h \subseteq g \Rightarrow y:\bot, z:A}\,{\scriptstyle RS.3}}{xR_{g:i}y, x:\mathcal{A}_{g:i}A, h \subseteq g \Rightarrow x:\mathcal{A}_{h:i}A, y:\bot}\,{\scriptstyle R\mathcal{A}_{h:i}}}{x:\mathcal{A}_{g:i}A, h \subseteq g \Rightarrow x:\mathcal{A}_{h:i}A, x:\mathcal{A}_{g:i}\bot}\,{\scriptstyle R\mathcal{A}_{g:i}} \qquad \dfrac{}{x:\bot,\ldots \Rightarrow x:\mathcal{A}_{h:i}A}\,{\scriptstyle L\bot}}{x:\sim\mathcal{A}_{g:i}\bot, x:\mathcal{A}_{g:i}A, h \subseteq g \Rightarrow x:\mathcal{A}_{h:i}A}\,{\scriptstyle L\supset}}{x:\sim\mathcal{A}_{g:i}\bot \,\&\, \mathcal{A}_{g:i}A, h \subseteq g \Rightarrow x:\mathcal{A}_{h:i}A}\,{\scriptstyle L\&}}{h \subseteq g \Rightarrow x:(\sim\mathcal{A}_{g:i}\bot \,\&\, \mathcal{A}_{g:i}A) \supset \mathcal{A}_{h:i}A}\,{\scriptstyle R\supset}$$

Axiom *Unanim* is easily derivable by rule *RS.4*.

Finally, by propositional steps, the derivation of *Mon* reduces to that of the sequent $h \subseteq g, x:\mathcal{A}_{h:i}\bot \Rightarrow x:\mathcal{A}_{g:i}\bot$, so we conclude by Lemma 3. □

By an adaptation of the method illustrated in the previous section, we can prove that the system **G3KA** has the same good structural properties as the basic propositional calculus **G3c** it is built upon. In particular, we have:

Theorem 5. *All the rules of* **G3KA** *are hp-invertible and the structural rules of weakening, contraction, and cut admissible. Weakening and contraction are in addition hp-admissible.*

Proof. Routine. □

Proposition 6. *The rules of modus ponens and necessitation are admissible in* **G3KA**.

Proof. If the sequents $\Rightarrow x : A$ and $\Rightarrow x : A \supset B$ are derivable in **G3KA**, then by invertibility of the right rule for implication we derive $x : A \Rightarrow x : B$ and by admissibility of cut we derive $\Rightarrow x : B$.

If $\Rightarrow w : A$ is derivable, then by substitution also $\Rightarrow y : A$ is derivable for an arbitrary label y, and by weakening also $xR_{g:i}y \Rightarrow y : A$ is derivable. A step of $R\mathcal{A}_{g:i}$ gives the conclusion $\Rightarrow x : R_{g:i}A$. □

Corollary 7. *The system* **G3KA** *is a complete sequent calculus for the logic of acceptance in the axiomatization of* [13].

4 Extensions with legislators

In this section we study extensions of the basic system. In particular, we consider rules that allow the possibility of *operative members* or *legislators* who can accept views for the group on behalf of other group members. The axiom for legislators considered in [13] is

$$\mathcal{A}_{g:i}(\bigwedge_{k\in Leg(i)} \mathcal{A}_{k:i}A \supset A) \quad \textbf{Leg}$$

where $Leg(i)$ is a finite non-empty set. We show that it corresponds to the frame property

$$\forall xy(xR_{g:i}y \supset \bigvee_{k\in Leg(i)} yR_{k:i}y) \quad \textbf{FLeg}$$

This property gives, for $Leg(i) \equiv \{k_1, \ldots, k_n\}$, the n-premiss rule

$$\frac{yR_{k_1:i}y, xR_{g:i}y, \Gamma \Rightarrow \Delta \quad \ldots \quad yR_{k_n:i}y, xR_{g:i}y, \Gamma \Rightarrow \Delta}{xR_{g:i}y, \Gamma \Rightarrow \Delta}\ \scriptstyle{RLeg}$$

We have:

Proposition 8. *The axiom for legislators is derivable in* **G3KA** *extended with rule* RLeg.

Proof. Starting root-first from the sequent to be derived, we have

$$\dfrac{\dfrac{\dfrac{\dfrac{\{xR_{g:i}y, yR_{k_j:i}y, y : \mathcal{A}_{k_1:i}A, \ldots, y : \mathcal{A}_{k_n:i}A \Rightarrow y : A\}_{j=1,\ldots,n}}{xR_{g:i}y, y : \mathcal{A}_{k_1:i}A, \ldots, y : \mathcal{A}_{k_n:i}A \Rightarrow y : A}\ \scriptstyle{RLeg}}{xR_{g:i}y, y : \bigwedge_{k\in Leg(i)} \mathcal{A}_{k:i}A \Rightarrow y : A}\ \scriptstyle{L\&}}{xR_{g:i}y \Rightarrow y : \bigwedge_{k\in Leg(i)} \mathcal{A}_{k:i}A \supset A}\ \scriptstyle{R\supset}}{\Rightarrow x : \mathcal{A}_{g:i}(\bigwedge_{k\in Leg(i)} \mathcal{A}_{k:i}A \supset A)}\ \scriptstyle{R\mathcal{A}_{g:i}}$$

where the n premisses of rule for legislators are indexed over the set $\{k_1, \ldots, k_n\}$ of members of $Leg(i)$; one step of $L\mathcal{A}_{k_j:i}$ produces the derivable sequents

$$\{xR_{g:i}y, yR_{k_j:i}y, y : A, y : \mathcal{A}_{k_1:i}A, \ldots, y : \mathcal{A}_{k_n:i}A \Rightarrow y : A\}_{j=1,\ldots,n}$$

□

By the above, rule *RLeg* is sufficient to derive the legislator axiom *Leg*. This means, indirectly, that the frame condition *FLeg* is sufficient to validate the legislator axiom. In order to show that it is characteristic we prove the following:

Proposition 9. *The frame condition* FLeg *holds in the canonical model for the logic of acceptance extended with the legislator axiom* Leg.

Proof. Recall that the canonical accessibility relation is defined by

$$xR_{k:i}y \equiv \text{ for all } A.x \Vdash \mathcal{A}_{k:i}A \text{ implies } y \Vdash A$$

Suppose that the antecedent of *FLeg*, $xR_{g:i}y$, holds. By validity of *Leg*, we have that $y \Vdash \bigwedge_{k \in Leg(i)} \mathcal{A}_{k:i}A \supset A$, that is,

$$\text{if } y \Vdash \bigwedge_{k \in Leg(i)} \mathcal{A}_{k:i}A, \text{ then } y \Vdash A$$

By unfolding the forcing relation on the conjunction, the above can be rewritten as

$$\text{if } \bigwedge_{k \in Leg(i)} y \Vdash \mathcal{A}_{k:i}A, \text{ then } y \Vdash A$$

Observe that the antecedent of this implication is a conjunction, so by the classical tautology $A\&B \supset C$ if and only if $(A \supset C) \vee (B \supset C)$, it can be rewritten as

$$\bigvee_{k \in Leg(i)} (y \Vdash \mathcal{A}_{k:i}A \rightarrow y \Vdash A)$$

Here the formula in parentheses is $yR_{k:i}y$, by arbitrariness of A and by the definition of the canonical accessibility relation, so we have proved that the frame condition

$$\forall xy(xR_{g:i}y \supset \bigvee_{k \in Leg(i)} yR_{k:i}y)$$

holds in the canonical model. □

Corollary 10. *The legislator axiom* Leg *is canonical with respect to the frame condition* FLeg.

Similarly, the requirement that legislators of an institution i must function as members of i, expressed in [13] by the principle

$$\sim \mathcal{A}_{Leg(i):i} \perp \quad \mathbf{Leg_o}$$

corresponds to the geometric frame condition

$$\forall x \exists y.\ xR_{Leg(i):i}y \quad \mathbf{FLeg_o}$$

which is turned into the rule

$$\frac{xR_{Leg(i):i}y, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta}\ {\scriptstyle RLeg_o}$$

with the condition that y is not in the conclusion.

In fact, we have:

Proposition 11. *The axiom* Leg_0 *is derivable in* **G3KA** *extended with rule* RLeg_0.

Proof. We have the following derivation, where the topsequent is an instance of $L\bot$:

$$\dfrac{\dfrac{\dfrac{y :\bot, xR_{Leg(i):i}y, x : \mathcal{A}_{Leg(i):i} \bot \Rightarrow x :\bot}{xR_{Leg(i):i}y, x : \mathcal{A}_{Leg(i):i} \bot \Rightarrow x :\bot}\, L\mathcal{A}_{Leg(i):i}}{x : \mathcal{A}_{Leg(i):i} \bot \Rightarrow x :\bot}\, RLeg_0}{\Rightarrow x :\sim \mathcal{A}_{Leg(i):i} \bot}\, R\supset$$

□

Conversely we have:

Proposition 12. *Any frame that validates axiom* Leg_0 *satisfies the frame condition* FLeg_0.

Proof. Observe that $\forall x. x \Vdash \sim \mathcal{A}_{Leg(i):i} \bot$ is classically equivalent to $\forall x \exists y.\ xR_{Leg(i):i}y$. □

Corollary 13. *Axiom* Leg_0 *is canonical with respect to the frame condition* FLeg_0.

The majority axiom can be dealt with in a similar way to the legislator axiom and a rule obtained by just replacing the set of legislators with the majority set in rule *RLeg*. However, extension of the logic with a majority principle may lead to inconsistent group views in situations exemplified by the *discursive dilemma* in which the views of the group members are distributed so that there is a majority for both a conclusion and premisses that entail the negation of the conclusion, see [12]. The discursive dilemma has been formalized using the logic of acceptance, and it was shown that it leads to an inconsistent view on the group level when a majority principle is used [1]. This can be shown using the sequent calculus system as well, but for lack of space we will not do that here. In addition to the majority rule leading to inconsistency at the group level, also legislator rules that allow determining a group view on the basis of a proper subset of the group members seem to face related problems: They may lead to an inconsistency at the level of individuals. This can be seen by constructing a case in which the legislators accept a proposition, say A, and some non-legislators accept its negation. By the legislator axiom, the group accepts A, and by axiom *Inc* we can then derive that all group members, even those who were against, accept the view A accepted by the legislators.

The problem does not appear with the *Unanim* rule that demands consensus among all group members. Even so, these problems seem to show that *Unanim* is not acceptable as an axiom, either. The purpose of axiom *Unanim* is to model the formation of a group view on the basis of consensus. Similarly, axioms *Leg* and *Maj* attempt to model the formation of a group view on the basis of majority voting or consensus among legislators, respectively. So the idea is to model collective decision-making, and the intuitive semantics of an acceptance operator $\mathcal{A}_{c:i}A$ would be something like "individual c votes for A as the group's view in context i".

However, the attempt to model formation of a group view clashes with the attempt to model what *follows* from the adoption of a view by a group. It is a generally accepted principle concerning group views that when a group accepts a view, then every group member accepts that view when operating as a member of the group. This idea is encoded in axiom *Inc*, but it does not fit with the intuitive semantics suggested above, because now we are speaking of individual acceptance *after* the formation of the group view whereas previously we were thinking about acceptance in the voting situation, that is, *before* the formation of the group view. These two senses of acceptance cannot be modelled simultaneously without either using different modalities for pre- and post-voting views, e.g., by using different context variables, or using some kind of a dynamic or temporal logic that allows changes in views. The reason that *Unanim* does not lead to inconsistent acceptances is that it requires that everyone agrees and thus nobody will have to change one's mind.

One will thus have to choose which aspect of collective acceptance one wants to model with the logic of acceptance: Focus either on what follows from existing group views or study the formation of group views. In the former case, one can have axioms *PAccess*, *NAccess*, *Inc*, and *Mon* but not axioms that derive group views from individual acceptances. In the latter case, one can have any axiom that allows deriving group views from individual, *Unanim*, *Leg* or *Maj*, but one should not then include axiom *Inc* that allows deriving individual views from the collective view.

5 Conclusion and future work

We have here presented a sequent calculus system for the logic of acceptance and proved the completeness of our system of sequent calculus with respect to an existing axiomatization of the logic. Because of the explicit use of labels, completeness with respect to the characterizing class of frames can also be established in a direct way following [16]: For every sentence of the logic we can either find a proof or a countermodel in the corresponding frame class. We can also show how the search space can be limited by methods of proof analysis in order to obtain decision procedures. Owing to the invertibility of the rules, cut-freeness, and bounded search space, our calculus permits to make conclusions not only about derivability but also about underivability of certain propositions and to study the sources of inconsistencies, which is not possible in the axiomatic approach. The methods presented can be adapted to the treatment of other non-summative collective attitudes that are based on collective acceptance beside group beliefs, for instance, group goals and collective preferences. This will be left for future work.

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