

# The Logic of Sense and Reference

Reinhard Muskens

Tilburg Center for Logic and Philosophy of Science (TiLPS)

ESSLLI 2009, Day 1

# The Logic of Sense and Reference

- In this course we look at the problem of the **individuation of meaning**.
- Many semantic theories do not individuate meanings finely enough and as a consequence make wrong predictions.
- We will discuss strategies to arrive at fine-grained theories of meaning. They will be illustrated mainly (though not exclusively) on the basis of my work.
- Strategies that can be implemented in standard higher order logic will be investigated, but generalisations of that logic that help deal with the problem will be considered too.
- **Today** I'll focus on explaining the problem itself and will mention some general strategies to deal with it. One of these (that of Thomason 1980) will be worked out in slightly more detail.

# What is Meaning? And what is Synonymy?

- What is meaning? The question is not easy to answer...

# What is Meaning? And what is Synonymy?

- What is meaning? The question is not easy to answer...
- But we can form **theories** of meaning.
- Lewis (1972):

*In order to say what a meaning **is**, we may first ask what a meaning **does**, and then find something that does that.*

- In today's talk I want to highlight some properties that meanings seem to have. If we want to find things that behave similarly they will need to have these properties too.
- In particular, I will look at the **individuation** of meaning. When are the meanings of two expressions identical? Or, in other words, **what is synonymy?**

# The Most Certain Principle

- Cresswell's (1982) Most Certain Principle:

*I'm going to begin by telling you what I think is the most certain thing I think about meaning. Perhaps it's the only thing. It is this. If we have two sentences A and B, and A is true and B is false, then A and B do not mean the same.*

- Meaning **determines** truth conditions.
- In Fregean terms, the **sense** of a sentence (a **thought**) determines its **reference** (a **truth value**).

# Compositionality

- Compositionality: The meaning of an expression is a function of the meanings of its parts.
- In order to **compute** the meaning of an expression, look up the meanings of the basic expressions forming it and successively compute the meanings of larger parts until a meaning for the whole expression is found.
- Compositionality at work in arithmetic: In order to compute the value of  $(x + y)/(z \times u)$ , look up the values of,  $x$ ,  $y$ ,  $z$ , and  $u$ , then compute  $x + y$  and  $z \times u$ , and finally compute the value of the whole expression.
- Many philosophers and linguists hold that Compositionality is at work in ordinary language too.

# Why Compositionality is Attractive

- Compositionality gives a nice **building block** theory of meaning.
- [Expressions [are [built [from [words [that [combine [into [[larger [and larger]] subexpressions]]]]]]]]]]
- In order to **compute** the meaning of an expression, look up the meanings of its words and successively compute the meanings of larger parts until a meaning for the whole expression is found.
- The theory **explains** how people can easily understand sentences they have never heard before, even though there are an infinite number of sentences any given person at any given time has not heard before.

# Frege on the Creativity of Meaning

Frege in his unpublished 'Logic in Mathematics' (1914):

*It is marvelous what language achieves. By means of a few sounds and combinations of sounds it is able to express a vast number of thoughts, including ones which have never been grasped or expressed by any human being. What makes these achievements possible? The fact that thoughts are constructed out of building-blocks. And these building-blocks correspond to groups of sounds out of which the sentence which expresses the thought is built, so that the construction of the sentence out of its parts corresponds to the construction of the thoughts out of its parts.*

Although this is not exactly the Compositionality principle, it seems to come close.

# Compositionality and the Congruence Principle

- Given reasonable assumptions Compositionality entails the **Congruence Principle**:
- Whenever  $A$  is part of  $B$  and  $A'$  means just the same as  $A$ , replacing  $A$  by  $A'$  in  $B$  will lead to a result that means just the same as  $B$ .
- - a. blah blah blah **such and such** blah blah
  - b. blah blah blah **so and so** blah blah
- If **such and such** and **so and so** mean the same thing, then a. and b. mean the same too.
- Conversely: if a. and b. do not mean the same, then **such and such** and **so and so** are not synonymous either.

## A Test for Synonymy

- Suppose we accept the Most Certain Principle (difference in truth-conditions implies difference in meaning) and the Congruence Principle (replacing synonyms by synonyms results in a synonymous expression). Then we have a **diagnostics** for synonymy:
- Replacing synonyms by synonyms preserves truth conditions, or
- If replacing  $A$  by  $B$  in some sentence  $C$  does not preserve truth conditions,  $A$  and  $B$  are not synonymous.
- But now, it will be shown, we are in dire straits. For it can be argued that there is no nontrivial synonymy...

# Examples of the Test Giving Unsurprising Results

- The **cat** is on the mat  
The **dog** is on the mat
- The sentences above differ in truth conditions. Hence **cat** and **dog** are not synonymous.
- John is a **Greek**  
John is a **Hellene**
- In this case there is no difference in truth conditions. But there might be another context that does give a difference...

# Contentious Cases

- a. Mary believes that John is a **Greek**  
b. Mary believes that John is a **Hellene**
- a. The Ancients knew that Hesperus was **Hesperus**  
b. The Ancients knew that Hesperus was **Phosphorus**
- In these cases most **language users** do perceive a difference in truth conditions while some **philosophers** vehemently deny that the a. sentences could be true in situations where the b. sentences are false.
- It is important here of course that the context of substitution is within the scope of a verb of **propositional attitude**.

# Hesperus and Phosphorus Again

- Consider the following two example sentences.
- Kripke holds that we do not know *a priori* that Hesperus is **Phosphorus**  
Kripke holds that we do not know *a priori* that Hesperus is **Hesperus**
- While the first sentence is true (of Kripke in **Naming and Necessity**), the second is certainly false.
- Conclusion: **Hesperus** and **Phosphorus** cannot be synonymous.
- But that is puzzling. . .

# Mates' Argument

- Mates (1952) used contexts in the scope of **two** attitude verbs to show that there is no non-trivial synonymy.
- Everybody believes that whoever thinks that all Greeks are courageous thinks that all **Greeks** are courageous  
Everybody believes that whoever thinks that all Greeks are courageous thinks that all **Hellenes** are courageous
- The first sentence presumably is true; the second is not.  
Conclusion: non-synonymy of **Greek** and **Hellene**.
- The argument can be repeated for any pair of purported synonyms (as long as they are not syntactically identical).

## (Non-)synonymy of Phrases

- The term ‘synonymy’ is usually used in the context of words, not of complex phrases. But if it is used as shorthand for ‘identity of meaning’ it may apply to larger phrases as well.
- Mary believes that **the dog is out if the cat is in**  
Mary believes that **the cat is out if the dog is in**
- These sentences might well differ in truth value. Conclusion: **the dog is out if the cat is in** and **the cat is out if the dog is in** are not identical in meaning.
- But the last two sentences are logically equivalent!
- Conclusion: Logically equivalent sentences can have different meanings.

# Taking Stock

- Given the Most Certain Principle and the Congruence Principle it can be shown that **no two syntactically different phrases can have the same meaning**.
- The Most Certain Principle is really hard to deny. The Congruence Principle follows from Compositionality, which seems very attractive.
- My position is that we should indeed bite the bullet, accept the principles and deal with the consequences.

## Two Problems

If we accept that there is no nontrivial synonymy we face at least two problems/tasks:

1. Give an account of the relation between pairs such as Greek/Hellene, ophtalmologist/eye doctor, die/kick the bucket, etc. If this is not synonymy proper, then what other relation is it?
2. If we want to model natural language meaning with the help of **logic** (and we do), we will need a logic in which it is possible to **distinguish between the values of logically equivalent expressions.**

# Today's Plan

- The first task mentioned on the previous slide (What is the **ordinary** relation of synonymy, if it is not strict identity of meaning?) is important and I think there may be good ways to deal with it, but in this course we will concentrate on the second task.
- We need a logic with a very **fine-grained** notion of meaning: Lots of distinctions between possible semantic values.
- In the following slides we will first see how the usual **possible worlds semantics** fails the requirements.
- Then we'll look at some alternative proposals that are on the market.
- And work out one in some more detail.

# Possible Worlds Semantics

- In possible worlds semantics the meaning of a sentence is identified with the set of worlds in which the sentence is true.
- Since **the dog is out if the cat is in** and **the cat is out if the dog is in** are true in exactly the same worlds, the theory predicts that these sentences are meaning-equivalent. But we have seen that this is incorrect. The argument can be repeated for any two co-entailing sentences.
- Many possible worlds theorists additionally hold (following Mill, Kripke, Donellan and others) that the meaning of a name simply is its bearer.
- This aggravates the problem since two names with the same bearer, such as Hesperus and Phosphorus, will then incorrectly be predicted to be substitutable for one another.

# Hintikka's Theory of Propositional Attitudes

- Within possible worlds semantics an influential theory of propositional attitude verbs is the one of Hintikka (1962): A person knows (believes) a proposition, if that proposition is true in all of that person's epistemic (doxastic) alternatives.
- I.e. 'John knows that\_' essentially is a modal  $\Box$  operator.
- A consequence of this theory is that belief and knowledge now are not only closed under logical equivalence, but also under **entailment**.
- Mary believes that the cat is out if the dog is in.  
Mary believes that the cat is in.  
Therefore, Mary believes that the dog is out.
- But this shouldn't hold. Mary may be an imperfect reasoner.

# Impossible Possible Worlds

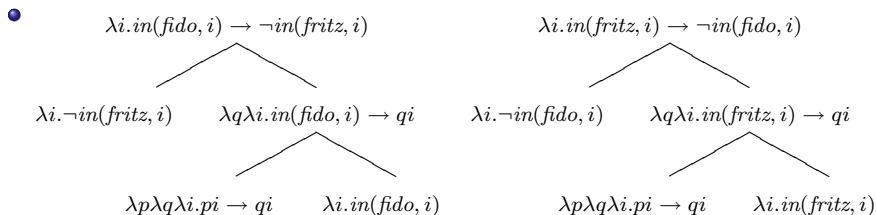
- The possible worlds approach does not make enough distinctions between meanings: In a sense there are too few sets of possible worlds.
- The situation can be remedied by throwing in more worlds, so that there will also be more sets of worlds.
- The logical operators need **not** have their usual meaning at these new points of reference and logical validities will therefore cease to hold throughout the set of all worlds.
- The name “impossible (possible) world” derives from Hintikka (1975), but the idea was also present in Montague (1970) and Cresswell (1972) and has been followed up in Rantala (1982), Muskens (1991), Barwise (1997), and Zalta (1997), to mention but a few.
- Tomorrow we’ll have a closer look at impossible possible worlds semantics.

# Structured Meanings

- Carnap (1947), who was the first to give what essentially amounted to a possible worlds analysis of natural language, also noticed problems with this account similar to the ones we have discussed.
- He therefore proposed a theory of **structured meanings**. Lewis (1972) and Cresswell (1985) have similar theories.
- In Lewis (1972) a **meaning** is ‘a finite ordered tree having at each node a category and an appropriate intension’.
- Here the ‘appropriate intension’ is just the intension the relevant subexpression gets in the ordinary possible worlds approach.
- I’ll illustrate this on the next slide, forgetting about the category labels.

# Structured Meanings: An Example

- [[Fritz is out] [if [Fido is in]]]
- [[Fido is out] [if [Fritz is in]]]



- Although the terms at the roots of these trees denote the same set of worlds, the trees themselves are distinct.
- Problems with distinguishing **all woodchucks are woodchucks** from **all woodchucks are groundhogs**: same trees, same intensions!

# Meanings as Algorithms

- Moschovakis (1994) proposes to identify Fregean senses with **algorithms** and references with the **values** returned by those algorithms.
- He formalizes the idea using a system called the *Lower Predicate Calculus with Reflection* (predicate logic + recursion).
- The idea that senses are procedures that can be used to compute reference is an old one, attributed to Frege in Dummett (1978).
- Frege's famous explanation of sense as the *Art des Gegebenseins* of a referent certainly can be read as expressing something closely akin to this point of view.
- Clearly, algorithms can be different and still have the same input-output conditions.
- **Wednesday** I'll discuss a theory directly inspired by Moschovakis' work and on **Thursday** there is of course Moschovakis' evening lecture. . .

# Propositions as Primitive Entities

- Thomason (1980) formulates an ‘Intentional Logic’ that is in fact a variant of Montague’s (1973) ‘Intensional Logic’.
- There is a primitive type  $p$  for propositions and propositions are taken to be **primitive**.
- A function sends propositions to their usual extensions (or, if so desired, to their usual intensions in the modal sense).
- The function need not be an injection: two or more propositions may go to the same value.
- It can be argued that Thomason’s theory gives the overall logical form of the kind of theories we have seen thus far: While other theories flesh out propositions in some way or other, Thomason’s gives a general account.

# A Streamlined Intentional Logic

- In the following we will work out a little theory along the lines of Thomason (1980).
- We will build upon Thomason's work and introduce a type  $p$  of primitive propositions.
- Unlike Thomason, we will work in a classical type logic with ground types  $e$  (for entities),  $s$  (possible worlds),  $p$  (propositions or senses), and  $t$  (truth-values).

# A Set of Non-logical Constants

Non-logical Constants	Type
not	$pp$
and, or, if	$p(pp)$
every, a, no, the	$(ep)((ep)p)$
is, love, kiss, ...	$e(ep)$
hesperus, phosphorus, mary, ...	$(ep)p$
planet, man, woman, run, ...	$ep$
necessarily, possibly	$pp$
believe, know, aware	$p(ep)$
<i>hesperus, phosphorus, mary, ...</i>	<i>e</i>
<i>love, kiss, ...</i>	<i>e(e(st))</i>
<i>planet, man, woman, ...</i>	<i>e(st)</i>
<i>acc</i>	<i>s(st)</i>
<i>believe, know, aware</i>	<i>p(e(st))</i>

# Some Terms of Type $p$

- ①  $((a \text{ woman})\text{walk})$
- ②  $((no \text{ man})\text{talk})$
- ③  $(\text{hesperus } \lambda x((a \text{ planet})(is \ x)))$
- ④  $((if((a \text{ woman})\text{walk}))((no \text{ man})\text{talk}))$
- ⑤  $((if((a \text{ man})\text{talk}))((no \text{ woman})\text{walk}))$
- ⑥  $(\text{mary}(\text{aware}((if((a \text{ woman})\text{walk}))((no \text{ man})\text{talk}))))$
- ⑦  $(\text{mary}(\text{aware}((if((a \text{ man})\text{talk}))((no \text{ woman})\text{walk}))))$
- ⑧  $((a \text{ woman})\lambda x(\text{mary}(\text{aware}((if(\text{walk } x))((no \text{ man})\text{talk}))))))$

- Consider the closed terms of type  $p$  built from constants in non-italic sans serif and variables, using application and linear abstraction. Denote this set with  $\mathcal{T}_{LF}^p$ .
- Since all the constants that are used in  $\mathcal{T}_{LF}^p$  are non-logical, there is really not much logic here. We can do  $\beta\eta$ -conversions and that is basically all.
- There is a close similarity between these terms and the usual LF trees. Compare e.g. the LF from Heim and Kratzer's textbook in (a) with the type  $p$  term (b).

(a)  $[_S[_{DP} \text{ every linguist}][_1[_S \text{ John}][_{VP} \text{ offended } t_1]]]]$

(b)  $((\text{every linguist})\lambda x_1(\text{john}(\text{offend } x_1)))$

# Some Meaning Postulates

- $r(\text{not } \pi) = \lambda i. \neg r\pi i$
- $r(\text{and } \pi\pi') = \lambda i. r\pi i \wedge r\pi' i$
- $r(\text{or } \pi\pi') = \lambda i. r\pi i \vee r\pi' i$
- $r(\text{if } \pi\pi') = \lambda i. r\pi i \rightarrow r\pi' i$
- $r(\text{every } \mathcal{P}'\mathcal{P}) = \lambda i. \forall x[r(\mathcal{P}'x)i \rightarrow r(\mathcal{P}x)i]$
- $r(\text{a } \mathcal{P}'\mathcal{P}) = \lambda i. \exists x[r(\mathcal{P}'x)i \wedge r(\mathcal{P}x)i]$
- $r(\text{no } \mathcal{P}'\mathcal{P}) = \lambda i. \neg \exists x[r(\mathcal{P}'x)i \wedge r(\mathcal{P}x)i]$
- $r(\text{necessarily } \pi) = \lambda i. \forall j[\text{acc } ij \rightarrow r\pi j]$
- $r(\text{possibly } \pi) = \lambda i. \exists j[\text{acc } ij \wedge r\pi j]$
- $r(\text{mary } \mathcal{P}) = r(\mathcal{P} \text{ mary})$  (and similarly for hesperus etc.)
- $r(\text{is } xy) = \lambda i. (x = y)$
- $r(\text{love } xy) = \text{love } xy$  (similarly for kiss, ...)
- $r(\text{planet } x) = \text{planet } x$  (similarly for man, woman, ...)
- $r(\text{believe } \pi x) = \text{believe } \pi x$  (similarly for try, wish, know, aware)

# From Propositions to Sets of Worlds

$$r((\text{if}((\text{a woman})\text{walk}))((\text{no man})\text{talk})) =$$

$$\lambda i. r((\text{a woman})\text{walk})i \rightarrow r((\text{no man})\text{talk})i =$$

$$\lambda i. \exists x[r(\text{woman } x)i \wedge r(\text{walk } x)i] \rightarrow r((\text{no man})\text{talk})i =$$

$$\lambda i. \exists x[r(\text{woman } x)i \wedge r(\text{walk } x)i] \rightarrow \neg \exists x[r(\text{man } x)i \wedge r(\text{talk } x)i] =$$

$$\lambda i. \exists x[\text{woman } xi \wedge \text{walk } xi] \rightarrow \neg \exists x[\text{man } xi \wedge \text{talk } xi]$$

- It is clear that  $r$  will send  $((\text{if}((\text{a woman})\text{walk}))((\text{no man})\text{talk}))$  and  $((\text{if}((\text{a man})\text{talk}))((\text{no woman})\text{walk}))$  to the **same** set of worlds.
- But it is consistent to assume that these terms denote **different** type  $p$  objects.

# Hyperintensionality

- $r(\text{mary}(\text{aware}(\text{if}(\text{a woman walk})(\text{no man talk})))) = \text{aware}(\text{if}(\text{a woman walk})(\text{no man talk}) \text{ mary}$
- $r(\text{mary}(\text{aware}(\text{if}(\text{a man talk})(\text{no woman walk})))) = \text{aware}(\text{if}(\text{a man talk})(\text{no woman walk}) \text{ mary}$
- There is no entailment, even though the embedded sentences co-entail.

# Conclusion

- We started with two principles that seem to govern the concept of meaning in natural language, the Most Certain Principle and the Principle of Congruence.
- With these two principles in hand it can be argued that the notion of synonymy must be very fine-grained indeed. If we want to model natural language meaning with the help of some logic, that logic must also make very fine distinctions.
- We proceeded to sketch some of the logical theories that are on the market and worked out a variant of an especially elegant one: the theory of Thomason (1980). More will follow!

# The Plan

- **Tomorrow** we will consider the strategy of obtaining finer granularity by **partializing** the higher order logic used for semantic description. We will also have a short look at an implementation of the **impossible possible worlds** approach in standard type theory.
- **Wednesday** we will work out Thomason's theory of primitive propositions further. I will argue that these primitive propositions can be made to behave much like Moschovakis' algorithms, given adequate meaning postulates. The underlying logic will be classical type theory.
- On **Thursday** and **Friday** I will discuss a **generalization** of type theory in which the axiom of **Extensionality** no longer holds. Without this axiom the logic becomes truly intensional, even before any possible worlds are introduced. It will be possible to reconstruct worlds as certain properties of propositions, though.

# References I

Barwise, J. (1997).

Information and Impossibilities.

*Notre Dame Journal of Formal Logic* 38(4), 488–515.

Carnap, R. (1947).

*Meaning and Necessity*.

Chicago: Chicago UP.

Cresswell, M. (1972).

Intensional Logics and Logical Truth.

*Journal of Philosophical Logic* 1, 2–15.

Cresswell, M. (1985).

*Structured Meanings*.

Cambridge, MA: MIT Press.

Dummett, M. (1978).

*Truth and Other Enigmas*.

London: Duckworth.

# References II

Hintikka, J. (1962).

*Knowledge and Belief.*

Cornell University Press.

Hintikka, J. (1975).

Impossible Possible Worlds Vindicated.

*Journal of Philosophical Logic* 4, 475–484.

Lewis, D. (1972).

General Semantics.

In D. Davidson and G. Harman (Eds.), *Semantics of Natural Language*, pp. 169–218. Dordrecht: Reidel.

Mates, B. (1952).

Synonymity.

In Linsky (Ed.), *Semantics and the Philosophy of Language*, pp. 111–136. Urbana: The University of Illinois Press.

## References III

Montague, R. (1970).

Universal Grammar.

*Theoria* 36, 373–398.

Reprinted in (Thomason 1974).

Montague, R. (1973).

The Proper Treatment of Quantification in Ordinary English.

In J. Hintikka, J. Moravcsik, and P. Suppes (Eds.), *Approaches to Natural Language*, pp. 221–242. Dordrecht: Reidel.

Reprinted in (Thomason 1974).

Moschovakis, Y. (1994).

Sense and Denotation as Algorithm and Value.

In *Logic Colloquium '90 (Helsinki 1990)*, Volume 2 of *Lecture Notes in Logic*, pp. 210–249. Berlin: Springer.

Muskens, R. (1991).

Hyperfine-Grained Meanings in Classical Logic.

*Logique et Analyse* 133/134, 159–176.

## References IV

Rantala, V. (1982).

Quantified Modal Logic: Non-normal Worlds and Propositional Attitudes.  
*Studia Logica* 41, 41–65.

Thomason, R. (Ed.) (1974).

*Formal Philosophy, Selected Papers of Richard Montague*.  
Yale University Press.

Thomason, R. (1980).

A Model Theory for Propositional Attitudes.  
*Linguistics and Philosophy* 4, 47–70.

Zalta, E. (1997).

A Classically-Based Theory of Impossible Worlds.  
*Notre Dame Journal of Formal Logic* 38(4), 640–660.