

The Rationality of Round Interpretation

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The issue of why there are round numbers has generally been analyzed using angles dealing with a variety of psychological and practical/strategic considerations (eg Dehaene and Mehler (1992), Wilson and Sperber (2002)). Expanding on a point made by Krifka (2007, p7-8), this paper uses elementary techniques from Bayesian statistics and game theory to reveal that in addition to such "outside" (and largely speaker-side) reasons, there are strong, purely a priori mechanisms pushing a rational listener towards assuming that round numbers are meant approximately.

Conditional Probability

Suppose k is the number of numbers that might be rounded to n , and suppose the (prior) odds that the speaker will round such a number to n are x . Four situations are relevant here, distinguished by whether or not the speaker intends the utterance roundly and whether the number referred to in question is n itself or merely sufficiently close. Assuming that the odds of any one number being the actual one are the same for all numbers under consideration -calling it p - the probabilities of these four are as follows:

	Actually n	Merely close to n
Rounded	xp	$x(k-1)p$
Not rounded	$(1-x)p$	$(1-x)(k-1)p$

Note that the bottom right situation will not involve an utterance of n . Thus, the chance that a given utterance of n is intended as a rounding is equal to

$$P(\text{round} | "n") = \frac{xp + (k-1)xp}{xp + (k-1)xp + (1-x)p} = \frac{kx}{kx + (1-x)} = \frac{k}{k-1 + \frac{1}{x}}$$

Hence, so long as the prior odds of rounding are fairly significant P will be close to one. For instance, if there is a 50% chance that people will round to the nearest multiple of ten (and, say, always round multiples of 5 up in those cases), then the chance that a multiple of ten is intended as a rounding is slightly over 90%. If there is a one in five chance for any number from 91 through 110 to be rounded to 100, then the chance that 100 is meant as such a rounding is over $\frac{5}{6}$. For a $\frac{1}{10}$ chance of rounding the nearest 200 numbers to 1000, P is over 95%.

Since these values for x seem underestimations if anything, we may conclude that it's generally statistically rational to assume that round numbers are intended as approximations. Of course, a speaker who knows that an utterance of a round number will be taken as approximate is likely to resort using a modifier such as "exactly" in those cases where an exact interpretation is intended (and sufficiently important). And this in turn will make a hearer dealing with an utterance that is not so modified even more justified in concluding that an approximation is intended. Over time, the de facto meaning of round numbers will evolve to be the approximate reading; in appropriate contexts, at any rate.

Game Theory

Under the simplifying assumption of equalizing the payoff for the speaker and the hearer, we may interpret this as a game of the form

	<i>round</i>	<i>not round</i>
<i>round</i>	a	b
<i>not round</i>	c	d

, with $a > b$ and $d > c$ (the row player is the speaker, so we need not have $a > c$ and $d > b$). However, in the "not round" case the number uttered can only be interpreted as round in one out of k cases (of the k numbers that might be rounded to n , only n itself can be interpreted as round). Thus we should replace c with $\frac{c}{k} + d\frac{k-1}{k} = d - \frac{d-c}{k}$, getting

	<i>round</i> [if a round number]	<i>not round</i>
<i>round</i>	a	b
<i>not round</i>	$d - \frac{d-c}{k}$	d

This makes round interpretation a good safety strategy if a , b , c and d are unclear, as one may expect $(a - b)$ to be a worse consequence for misinterpretation than $\frac{d-c}{k}$. It also makes that it is rational iff

$$x(a - b) > (1 - x)\frac{d - c}{k}$$

Which is to say

$$x > \frac{(d - c)}{(a - b)k + (d - c)}$$

Thus if, say, $(d - c) = (a - b)$ then x need only be as little as $\frac{1}{k+1}$ for round interpretation to be the rational choice.

Preferred disambiguation

(Some more elaborate analyses beyond the scope of this abstract.)

Discussion

These points don't of themselves offer sufficient explanation for why there is any roundness to begin with, especially without a good speaker-side argument, but they can provide leverage for such arguments by ensuring that only a modest prior chance of rounding needs to be explained.

Generalization of the arguments and results from this paper into observations on loose use of non-number words is also discussed, yielding possible explanations for the gradual loosening of extreme degree terms¹ and the origin of (some) vagueness.

Refs Krifka (2007), 'Approximate interpretation of number words: A case for strategic communication', *Cognitive foundations of interpretation*; Wilson and Sperber (2002), 'Truthfulness and Relevance', *Mind*; Dehaene and Mehler (1992), 'Cross-linguistic regularities in the frequency of number words', *Cognition*; Jansen and Pollmann (2001), 'On Round Numbers: Pragmatic Aspects of Numerical Expressions', *Journal of Quantitative Linguistics*; Ito and Tagliomonte (2003), 'Well weird, right dodgy, very strange, really cool: Layering and recycling in English intensifiers', *Language in Society*.

¹reported to be the origin of *very* by Ito and Tagliomonte (2003) and many others