

# Reasoning about permitted announcements (extended abstract)

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## 1 Introduction

Four players are playing the french game "la belote". All the 32 cards are dealt, each player starts the game with eight cards and she knows them, and all of this is common knowledge. We can formalize this situation using epistemic logic (see [8] for more details): the set of propositional atoms of our language is  $\{(VC)_i | V \in \{7, 8, 9, 10, J, Q, K, A\}, C \in \{\clubsuit, \heartsuit, \diamondsuit, \spadesuit\}, i \in \{1, 2, 3, 4\}\}$ . The model of "la belote" has four epistemic (equivalence) relations, a world corresponds to a deal, and we say that  $\mathcal{M}, s \models (10\spadesuit)_3$  if player 3 has the ten of spades in the deal  $s$ . Then we can express sentences like "player 2 knows that player 3 has the queen of hearts" ( $K_2(Q\heartsuit)_3$ ). What is happening when the game starts? Player one playing the 7 of diamonds can be seen as the public announcement that player one has the 7 of diamonds. But she may be able to do such an announcement, because she actually has the 7 of diamonds, without having the permission to do it, because the rules of the game may forbid it to her. Which are the announcements that are permitted in this game, depending on the actual situation? This paper will try to answer this question.

Our goal is to formalize the concept of "having the permission to say". The way we choose is to adapt the semantics considered in [11] in the particular case where actions are replaced by public announcements. Thus, for  $\alpha$  in van der Meyden's  $\diamond(\alpha, \varphi)$  we take an announcement  $\psi!$  such that  $\diamond(\psi!, \varphi)$  now means 'it is permitted to announce  $\psi$ , after which  $\varphi$  is true'. The primitive of our language is in fact somewhat different, as it also takes sequences of announcements into account, but we will be able to define such  $\diamond(\psi!, \varphi)$  as a special case (see section 4.1).

The logic of public announcements (*PAL*) proposed by Plaza in [9], which is an extension of epistemic logic, permits to express the way agents update their knowledge after public announcements of true propositions. We can for example write in this language  $\langle\psi\rangle\varphi$  which means that after the truthful public

announcement of  $\psi$ ,  $\varphi$  becomes true. This logic has been largely studied (see [7]) and extended (see for example [2] and [3]).

To speak about "the permission to say", we will extend *PAL* by introducing an operator  $P$  of permission,  $P(\psi, \varphi)$  expressing the fact that "after the announcement of  $\psi$  it is permitted to say  $\varphi$ ". We will first present the syntax and the semantics of our logic, and then we will prove some technical results, in particular the completeness of the axiomatisation and the decidability of the problem of satisfiability.

All the omitted proofs can be found in [4].

## 2 Logic of permission and public announcements

### 2.1 Syntax of $\mathcal{L}_{ppal}$

The language  $\mathcal{L}_{ppal}$  over a countable set of agents  $N$  and a countable set of propositional atoms  $\Theta$  is defined as follows:

$$\varphi ::= \perp | p | \neg\varphi | \psi \vee \varphi | K_i\varphi | [\psi]\varphi | P(\psi, \varphi)$$

where  $i \in N$  and  $p \in \Theta$ .

The intuitive reading of  $K_i\varphi$  is "agent  $i$  knows that  $\varphi$  is true" whereas  $[\psi]\varphi$  is read as "after  $\psi$  has been publicly announced, it is true that  $\varphi$ ". We read  $P(\psi, \varphi)$  as "after that  $\psi$  has been publicly announced, it is permitted to say  $\varphi$ ". We make use of the classical abbreviations for the other Boolean operators. Moreover, we have an operator  $\langle\psi\rangle$  defined by  $\langle\psi\rangle\varphi = \neg[\psi]\neg\varphi$ .

We call  $\mathcal{L}_{pel}$  the fragment without announcement operator and  $\mathcal{L}_{el}$  the fragment restricted to the Boolean and epistemic operators.

### 2.2 Semantics

The models of our logic are structures of the form  $\mathcal{M} = (S, \{\sim_i\}_{i \in N}, V, \mathcal{P})$  with  $S$  being a non-empty set of states,  $\sim_i$  being an equivalence relation between states of  $S$ ,  $V$  mapping propositional atoms to subset of  $S$  and  $\mathcal{P} \subseteq S \times 2^S \times 2^S$ . If the equivalence relation  $\sim_i$  holds between states  $s, t \in S$ , this means that, as far as agent  $i$  is concerned,  $s$  and  $t$  are indiscernible.

The membership of  $(s, S', S'')$  in  $\mathcal{P}$  can be interpreted as follows: in state  $s$ , any announcement that restricts the set of all possible states to  $S'$  will do so in such a way that any further announcement that restricts the set of all possible states to  $S''$  will become permitted.

We define the update of a model  $\mathcal{M}$  after the public announcement of  $\psi$  as the restriction  $\mathcal{M}_\psi$ :

**Definition 1 (restricted model)** For any model  $\mathcal{M}$  and any  $\psi \in \mathcal{L}_{ppal}$ , we define the restriction  $\mathcal{M}_\psi = (S_\psi, \sim_i^\psi, V_\psi, \mathcal{P}_\psi)$  where:

- $S_\psi = \llbracket \psi \rrbracket_{\mathcal{M}} = \{s \in S \mid \mathcal{M}, s \models \psi\}$

- for all  $p \in \Theta$ ,  $V_\psi(p) = V(p) \cap S_\psi$
- for all  $i$ ,  $\sim_i^\psi = \sim_i \cap (S_\psi \times S_\psi)$
- $\mathcal{P}_\psi = \{(s, S', S'') \in \mathcal{P} \mid s \in S_\psi, S' \subseteq S_\psi, S'' \subseteq S_\psi\}$

We can now define the satisfiability relation  $\models$ .

**Definition 2 (satisfiability relation)** Let  $\mathcal{M}$  be a model and  $s$  be a state of  $S$ . The satisfiability relation  $\mathcal{M}, s \models \varphi$  is defined inductively on the structure of  $\varphi$ :

$$\mathcal{M}, s \models p \text{ iff } s \in V(p)$$

$$\mathcal{M}, s \not\models \perp$$

$$\mathcal{M}, s \models \neg\psi \text{ iff } \mathcal{M}, s \not\models \psi$$

$$\mathcal{M}, s \models \psi_1 \vee \psi_2 \text{ iff } (\mathcal{M}, s \models \psi_1 \text{ or } \mathcal{M}, s \models \psi_2)$$

$$\mathcal{M}, s \models K_i\psi \text{ iff for all } t \sim_i s, \mathcal{M}, t \models \psi$$

$$\mathcal{M}, s \models [\psi]\chi \text{ iff } (\mathcal{M}, s \models \psi \Rightarrow \mathcal{M}_\psi, s \models \chi)$$

$$\mathcal{M}, s \models P(\psi, \chi) \text{ iff } (s, \llbracket \psi \rrbracket_{\mathcal{M}}, \llbracket \langle \psi \rangle \chi \rrbracket_{\mathcal{M}}) \in \mathcal{P}$$

For all  $\varphi \in \mathcal{L}_{ppal}$  we note  $\mathcal{M} \models \varphi$  iff for all  $s \in S$ ,  $\mathcal{M}, s \models \varphi$ . As usual, we note  $\models \varphi$  iff for all models  $\mathcal{M}$  we have  $\mathcal{M} \models \varphi$ .

We can define a function  $deg$  from  $\mathcal{L}_{ppal}$  to  $\mathbb{N}$  and a function  $tr$  from  $\mathcal{L}_{ppal}$  to  $\mathcal{L}_{pel}$  such that for all  $\varphi \in \mathcal{L}_{ppal}$  we have  $\models \varphi \leftrightarrow tr(\varphi)$  and  $deg(\psi) = deg(tr(\varphi))$ . This tools are very useful for many proofs that are omitted in this extended abstract.

An interesting property of our semantics is that the following proposition is true:

**Proposition 3** For all models  $\mathcal{M}$  and all formulas  $\psi, \psi', \varphi, \varphi' \in \mathcal{L}_{ppal}$  we have if  $\mathcal{M} \models (\psi \leftrightarrow \psi') \wedge (\langle \psi \rangle \varphi \leftrightarrow \langle \psi' \rangle \varphi')$  then  $\mathcal{M} \models P(\psi, \varphi) \leftrightarrow P(\psi', \varphi')$

It comes from the definition of the restriction  $\mathcal{P}_\psi$  and corresponds to the intuitions that

1. after two equivalent announcements the same formulas are permitted to be said and
2. if two sentences are equivalent, they are permitted to be said in the same way.

Table 1: The axiomatisation  $PPAL$

$K_i\varphi \rightarrow \varphi$	truth
$K_i\varphi \rightarrow K_iK_i\varphi$	positive introspection
$\neg K_i\varphi \rightarrow K_i\neg K_i\varphi$	negative introspection
$[\psi]p \longleftrightarrow (\psi \rightarrow p)$	atomic permanence
$[\psi]\perp \longleftrightarrow \neg\psi$	ann. and false
$[\psi]\neg\varphi \longleftrightarrow (\psi \rightarrow \neg[\psi]\varphi)$	ann. and negation
$[\psi](\varphi_1 \vee \varphi_2) \longleftrightarrow ([\psi]\varphi_1 \vee [\psi]\varphi_2)$	ann. and disjunction
$[\psi]K_i\varphi \longleftrightarrow (\psi \rightarrow K_i[\psi]\varphi)$	ann. and knowledge
$[\psi_1][\psi_2]\varphi \longleftrightarrow [(\psi)_1\psi_2]\varphi$	ann. composition
$[\psi]P(\psi', \varphi) \longleftrightarrow (\psi \rightarrow P(\langle\psi\rangle\psi', \varphi))$	ann. and permission

### 2.3 Soundness and completeness

Let  $PPAL$  be the smallest normal logic in our language that contains the schemata in table 1 and that is closed under the following inference rule:

- From  $(\psi \leftrightarrow \psi') \wedge (\langle\psi\rangle\varphi \leftrightarrow \langle\psi'\rangle\varphi')$  infer  $P(\psi, \varphi) \leftrightarrow P(\psi', \varphi')$

**Proposition 4** *PPAL is sound in all the Kripke-models where  $\sim_i$  are equivalence relations.*

To prove the completeness result, let us define the canonical model for  $PPAL$ :

**Definition 5 (Canonical Model)** *The canonical model  $\mathcal{M}^c = (S^c, \sim_i^c, V^c, \mathcal{P}^c)$  is defined as follows:  $S^c$  is the set of all  $\vdash_{PPAL}$ -maximal consistent sets; for any  $p \in \Theta$ ,  $V^c(p) = \{x \in S^c \mid p \in x\}$ ;  $x \sim_i^c y$  iff  $K_ix = K_iy$ , where  $K_ix = \{\varphi \mid K_i\varphi \in x\}$  and  $\mathcal{P}^c = \{(x, S', S'') \mid \exists P(\psi, \varphi) \in x \mid S' = \{y \in S^c : \psi \in y\}, S'' = \{y \in S^c : \langle\psi\rangle\varphi \in y\}\}$ .*

Then we have:

**Proposition 6 (Truth Lemma for  $\mathcal{L}_{pel}$ )** *For all  $\varphi \in \mathcal{L}_{pel}$  we have:*

$$\Pi(\varphi) : \text{for all } x \in S^c, \mathcal{M}_c, x \models \varphi \text{ iff } \varphi \in x$$

**Proposition 7** *PPAL is sound and complete with respect to the class of models where  $\sim_i$  are equivalence relations.*

**Proof** The soundness has been shown in Proposition 4. By Proposition 6 we can show the completeness with respect to the class of models where  $\sim_i$  are equivalence relations. Indeed, for all  $\varphi \in \mathcal{L}_{ppal}$ :  $\models \varphi \Rightarrow \models tr(\varphi) \Rightarrow \mathcal{M}_c \models tr(\varphi) \Rightarrow \vdash_{PPAL} tr(\varphi) \Rightarrow \vdash_{PPAL} \varphi$ .  $\square$

### 3 Case study

Let us take the example of the french game "la belote". After the deal, and after the choice of a trump suit, the first player plays any card of her hand (its color will be the *dominant* color), and then each player plays consecutively following the clockwise order. The player who played the highest card of trump or the highest at the same color of the first player's card wins the round and starts the following round, with any card of her hand. With the exception of the first player of a round, each player has to follow the asked color or, if she cannot, to play a trump. Moreover, when a trump is played, it is forbidden to play a lower trump. The whole rule of this game is explained in [1].

As we said before, we can see the act of playing a card as the public announcement that the corresponding card belongs to the corresponding player. In the language that we construct, the set of propositional atoms  $\Theta$  is  $\{(VC)_i \mid V \in \{7, 8, 9, 10, J, Q, K, A\}, C \in \{\clubsuit, \heartsuit, \diamondsuit, \spadesuit\}, i \in \{1, 2, 3, 4\}\}$ . Each atom  $(VC)_i$  has a value  $V$ , a color  $C$  and belongs to a player  $i$ . Recall that  $\mathcal{M}, s \models (VC)_i$  means that that in the deal  $s$ , the player  $i$  has the  $V$  of  $C$ . The trump suit has been selected before the game starts, we will suppose here that it is clubs. The set of atoms is partially ordered in the following way, which corresponds to the order of the cards in the game, where "\*" can be replaced by any player name.

For non-trumps (i.e. for any  $C \neq \clubsuit$ ):  $7C_* < 8C_* < 9C_* < JC_* < QC_* < KC_* < 10C_* < AC_*$ . For trumps:  $7\clubsuit_* < 8\clubsuit_* < Q\clubsuit_* < K\clubsuit_* < 10\clubsuit_* < A\clubsuit_* < 9\clubsuit_* < J\clubsuit_*$ .

The whole formalization of the game would be too long and useless for our purpose, but let us formalize, as an example, some of the rules of the game that are valid at the beginning of each round of the game. This condition implies that the models  $\mathcal{M}$  considered here are models in which the players have the same number of cards in their hands.

#### 1- One player at once:

For all  $\psi \in \mathcal{L}_{ppal}$ , all players  $i \neq j$ , all  $p_i, q_j \in \Theta$ ,  $\mathcal{M} \models P(\psi, p_i) \rightarrow \neg P(\psi, q_j)$ .

#### 2- Each card is played only once:

For all  $p \in \Theta$ , all  $\psi \in \mathcal{L}_{ppal}$ ,  $\mathcal{M} \models \neg P(p \wedge [p]\psi, p)$

#### 3- If you can play a card of the dominant color, it is forbidden not to do so :

For all values  $V_1, V_2, V'$ , all colors  $C \neq C'$ , all players  $i \neq j$ ,  
 $\mathcal{M} \models (V_1C)_j \rightarrow \neg P((V_2C)_i, (V'C')_j)$

#### 4- If you can play a trump, it is forbidden to play a card which is neither a trump neither of the dominant color:

For all values  $V_1, V, V'$ , all colors  $C \neq C'$  where  $C, C' \neq \clubsuit$ , all players  $i \neq j$ ,  $\mathcal{M} \models (V_1\clubsuit)_j \rightarrow \neg P((VC)_i, (V'C')_j)$

**5- Permission to say "belote et rebelote":** (The only exception to the next rule) For all player  $i$ ,

$$\mathcal{M} \models (K\clubsuit)_i \wedge (Q\clubsuit)_i \wedge P(\psi, (Q\clubsuit)_i) \rightarrow P(\psi, (Q\clubsuit)_i \wedge (K\clubsuit)_i)$$

**6- Forbidden to speak about the game:**

For all  $\psi, \varphi \in \mathcal{L}_{ppal}$ , for all state  $s$ ,  $\mathcal{M}, s \models P(\psi, \varphi)$  implies that  $\varphi \in \Theta$ .

**7- If you can play overtrump, it is forbidden to play a lower trump:**

For all  $\psi \in \mathcal{L}_{ppal}$ , all players  $i, j$ , all values  $V, V_1, V_2$  such that  $(V_1\clubsuit)_j < (V\clubsuit)_j < (V_2\clubsuit)_j$ ,  $\mathcal{M} \models (V_2\clubsuit)_j \rightarrow \neg P(\langle \psi \rangle (V\clubsuit)_i, (V_1\clubsuit)_j)$

Let us see the consequence of these conditional rules in the permission to speak of the following state (deal)  $s$ , where each player has 2 cards

$$\begin{array}{c}
 \text{Bill}(B) \\
 \boxed{Q\heartsuit} \boxed{K\spadesuit} \\
 \\
 \text{Anne} \boxed{7\heartsuit} \qquad \boxed{Q\clubsuit} \text{ Charles} \\
 (A) \boxed{8\heartsuit} \qquad \boxed{K\clubsuit} \quad (C) \\
 \\
 \boxed{8\clubsuit} \boxed{A\clubsuit} \\
 \text{Diane}(D)
 \end{array}$$

Anne has to start. According to the rule, our model validates the following formulas:

$\mathcal{M}, s \models P(8\heartsuit)_A \wedge P(7\heartsuit)_A \wedge \neg P((8\heartsuit)_A \wedge (7\heartsuit)_A)$ : Anne has the permission to play one of her cards, but not both (rule (6))

$\mathcal{M}, s \models \neg P((8\heartsuit)_A, (K\spadesuit)_B)$ : If Anne plays the  $8\heartsuit$ , Bill has not the permission to play another color (rule (3))

$\mathcal{M}, s \models P(\langle (8\heartsuit)_A \rangle (Q\heartsuit)_B, (Q\clubsuit)_C \wedge (K\clubsuit)_C)$ : When he has to play, Charles has the permission to announce that he has both cards of the "belote" (rule (5))

$\mathcal{M}, s \models \neg P(\langle \langle (8\heartsuit)_A \rangle \rangle (Q\heartsuit)_B) (Q\clubsuit)_C, (8\clubsuit)_D$ : After Charles played a trump, Diane can play a higher trump, so she has not the permission to play a lower one (rule (7)).

## 4 Comparison and further research

### 4.1 Comparison to the literature

Our logic to reason about permitted announcements can be seen as a continuation of the efforts begun in van der Meyden's [11] and Pucella and Weismann's [10]. Van der Meyden's notion of permission is definable in terms of ours. He assumes a set of action variables  $a$  in his logic. In our case, these actions are all announcements  $\varphi!$ . Employing that terminology, the correspondence is:

**Proposition 8**  $\diamond(\varphi!, \theta)$  is equivalent to  $P(\top, \varphi) \wedge \langle \varphi \rangle \theta$

Of course, van der Meyden's  $\diamond(\varphi!, \theta)$  having a linguistic version  $Perm(\varphi)\theta$  in [10], the relation between  $P(\top, \varphi)$  (ours) and  $Perm(\varphi)$  (Pucella and Weismann) is therefore even closer.

## 4.2 Dynamics of permission

Dynamics of permission have been studied by Pucella and Weismann in [10]. They propose to add new permissions by linking states which satisfy a proposition  $\rho_1$  to states which satisfy a proposition  $\rho_2$ , employing an operator called  $grant(\rho_1, \rho_2)$ .

A natural adaptation of their proposal, in the context where the actions are public announcements, is obtained by defining the modal operator  $grant(\rho_1, \rho_2)$  in the following way: for all models  $\mathcal{M}$  and all  $s \in \mathcal{M}$  we have:  $\mathcal{M}, s \models grant(\rho_1, \rho_2)\theta$  iff  $\mathcal{M}^g, s \models \theta$  where  $\mathcal{M}^g$  is as  $\mathcal{M}$  except for the permission relation which is expanded in the following way:  $\mathcal{P}^g = \mathcal{P} \cup \{(s', S_1, S_2) \mid s' \in S_2 \subseteq S_1, \mathcal{M}|_{S_1}, s' \models \rho_1 \text{ and } \mathcal{M}|_{S_2}, s' \models \rho_2\}$ . The correspondent *revoke* operator would be such that  $\mathcal{P}$  becomes  $\mathcal{P} \setminus \{(s', S_1, S_2) \mid s' \in S_2 \subseteq S_1, \mathcal{M}|_{S_1}, s' \models \rho_1 \text{ and } \mathcal{M}|_{S_2}, s' \models \rho_2\}$ .

Such a definition is particularly interesting if we think that the permission to say something depends on the consequences of our announcements.

But this natural variant is not appropriate in our case study. Indeed, we would like to be able to add a new rule of the game (or delete one), in which the permission to say something depends on the characteristic of the announcement itself, and not on the characteristics of the states that are linked by such an announcement.

To express this kind of permission granting, we define these other modal operators *GRANT* and *REVOKE*.

For all  $\chi, \psi, \varphi \in \mathcal{L}_{ppal}$ , let  $GRANT(\chi, P(\psi, \varphi))$  be such that for all models  $\mathcal{M}$  and all  $s \in \mathcal{M}$  we have:  $\mathcal{M}, s \models GRANT(\chi, P(\psi, \varphi))\theta$  iff  $\mathcal{M}^G, s \models \theta$  where  $\mathcal{M}^G$  is as  $\mathcal{M}$  except for the permission relation which is expanded in the following way:  $\mathcal{P}^G = \mathcal{P} \cup \{(s, \llbracket \psi \rrbracket_{\mathcal{M}}, \llbracket \langle \psi \rangle \varphi \rrbracket_{\mathcal{M}}) \mid s \in \llbracket \chi \rrbracket_{\mathcal{M}}\}$ . Intuitively, after applying  $GRANT(\chi, P(\psi, \varphi))$  the following becomes law: "if  $\chi$  is true, then after announcing  $\psi$  it is permitted to announce  $\varphi$ "

**Proposition 9** *Formula schema is valid:  $\models GRANT(\chi, P(\psi, \varphi))(\chi \rightarrow P(\psi, \varphi))$  (after granting that 'after announcing  $\varphi$ , it is permitted to announce  $\psi$ ', if  $\chi$  is true then it is true that after announcing  $\varphi$ , it is permitted to announce  $\psi$ )*

**Proof** This easy proof is left to the reader. □

We can introduce in a similar way the modal operator  $REVOKE(\chi, P(\psi, \varphi))$ : by applying it,  $\mathcal{P}$  becomes  $\mathcal{P}^R = \mathcal{P} \setminus \{(s, \llbracket \psi \rrbracket_{\mathcal{M}}, \llbracket \langle \psi \rangle \varphi \rrbracket_{\mathcal{M}}) \mid s \in \llbracket \chi \rrbracket_{\mathcal{M}}\}$ .

With both definitions of "grant" and "revoke" modal operators, the axiomatisation issue and the decidability issue remain open.

Apart from announcements, we can expand the language to involve assignments, or even more complex public informative events. A typical example is where the Lord Mayor (or relevant official, dependent on where you are...) announces a couple husband and wife. By changing 'the world', so to speak, they are now entitled to all rights and favours resulting from marriage. A lot more is permitted than before! This does not result from changing the law, but merely from applying the law to a new instance, this particular couple, as a result of changing the state of the world. Changes in the world can be obtained by public assignments, and logics for public announcement and public assignment [6] (or more, as already said) can thus in the obvious way be extended with permission operators.

### 4.3 Quantifying over permission

Suppose I am playing at "la belote". I can ask myself, in a given situation, if there is any card I can play such that I will win the current round. Such quantification over permitted announcement could be expressed introducing the following operator:  $\Diamond(\psi, !)\theta$  which express that  $P(\psi, \varphi) \wedge [\psi]\langle\varphi\rangle\theta$  for some  $\varphi$ . In other words, after the announcement of  $\psi$  there exists a permitted announcement such that after announcing it  $\theta$  becomes true. It is possible to adapt easily the works of [2] and [3] to define a sound and complete axiomatisation for this new language, with respect to the same class of models. A different form of quantification would be as in  $P(!, \varphi)$ , which expresses that  $P(\psi, \varphi)$  for any  $\psi$ . In other words, no matter what the further informative developments are,  $\varphi$  will *still* be permitted. One could think of basic human rights as falling in this category, as they cannot be outlawed by any foreseeable development. And into the 'forbidden no matter what' category fall things like 'thou shalt no kill'.

### 4.4 Obligation

To define the obligation to say something, we could add a new modal operator  $O(\psi, \varphi)$ , read as "after the announcement of  $\psi$ , it is obligatory to say  $\varphi$ " and a semantics analog to the previous, *i.e.* a set  $\mathcal{O}$  such that  $\mathcal{M}, s \models O(\psi, \varphi)$  iff  $(s, \llbracket\psi\rrbracket_{\mathcal{M}}, \llbracket\langle\psi\rangle\varphi\rrbracket_{\mathcal{M}}) \in \mathcal{O}$ . Just as before,  $O(\varphi)$  would be an abbreviation of  $O(\top, \varphi)$ .

Assume we have this definition, how to express the obligation for agent  $i$  to play the king or the queen of clubs, which corresponds to a situation in which it is obligatory to play trump? Note that  $O((K\clubsuit)_i \vee (Q\clubsuit)_i)$  does not express it, neither  $O(K\clubsuit)_i \vee O(Q\clubsuit)_i$ . It would be impossible to express the obligation to say  $\varphi_1$  or to say  $\varphi_2$ .

We tentatively suggest the following approach. Let us define the announcement  $\varphi!$  as an epistemic action, and so define the non-deterministic choice as usual. The obligation to say  $\varphi_1$  or to say  $\varphi_2$  would be expressed by  $O(\top, \varphi_1! \cup \varphi_2!)$ . The models of our logic would be tuples  $\mathcal{M} = (S, V, \sim_i, \mathcal{P}, \mathcal{O})$  where  $\mathcal{O} \subseteq S \times 2^S \times 2^{2^S}$ . We could then define the satisfiability relation inductively as before with for all  $\mathcal{M}$ , for all  $s \in S$ , for all  $\psi, \varphi_1, \dots, \varphi_n \in \mathcal{L}_{ppal}$  we have:  $\mathcal{M}, s \models O(\psi, \varphi_1! \cup \dots \cup \varphi_n!)$  iff  $(s, \llbracket\psi\rrbracket_{\mathcal{M}}, \{\llbracket\langle\psi\rangle\varphi_1\rrbracket, \dots, \llbracket\langle\psi\rangle\varphi_n\rrbracket\}) \in \mathcal{O}$ .



We did not consider such semantics in the definition of permission because we implicitly assumed that we add free-choice semantics, but actually the question of the alternative between free-choice or imposed-choice semantics remains open (for more details about these notions see [5]).

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