

Logicality and Invariance

When designing a logical system, one has to start by distinguishing between logical expressions, which are the proper objects under study and whose interpretations remain fixed across models, and non-logical expressions, whose interpretations are permitted to vary from one model to the other. In particular, this distinction is crucial for the definition of logical truth as truth in all models. But are there principled ways to draw the line between logical and non-logical expressions? Consider any logical system, what is its natural repertoire of logical operations?

This topic – the analysis of logicality – is typically an interdisciplinary topic at the crossroads of logic, linguistics and philosophy. Linguists would like to know what is special with 'grammatical' words, such as determiners, that get captured by logical expressions. Logicians would be like to understand what is special about the operations they study. And the very generality of these questions makes for their philosophical interest regarding theoretical foundations of logic and grammar.

In the model-theoretic tradition, invariance under permutation has long been the standard answer. It has been widely used in abstract model theory and in generalized quantifier theory. Tarski and Sher gave philosophical arguments in favor of it. Recent developments, however, suggest that this criterion is not fully satisfactory, and that a whole family of alternative criteria are well-worth considering. The aim of the course will be to review these various answers, show how they integrate in a common framework and evaluate their consequences with respect to general questions in logic (such as functional completeness results), in linguistics (such as the semantic nature of 'functional' or 'grammatical' expressions) and in the philosophy of the formal sciences (why is logic different from mathematics?). We will also look at the question of logicality in other logical settings (discussing the case of modal and dynamic logic) and from different perspectives (presenting the proof-theoretic approaches to the issue, and possible connections with invariance).

Day 1. *Background and main idea* ([6], [8], [9], [10])

*** Introducing the logicality question:

logical *vs* non-logical symbols in logic, functional *vs* lexical expressions in linguistics.

*** The standard approach in terms of invariance under permutation.

Day 2 to Day 4. *Logicality as invariance under X for various X s* ([1], [2], [3], [5])

*** Alternative criteria:

invariance under homomorphism (Feferman), invariance under potential isomorphism.

*** A unifying framework for invariance criteria:

The Galois connection for invariance

*** Invariance and Functional completeness

*** A case study:

Bisimulation and invariance for modal and dynamic logic

Day 5. *Alternative approaches* ([4], [7])
*** Harmony and other ideas from Dummett
*** Sambin's definitional equations.
*** The quest for a unified perspective.

References

- [1] J. van Benthem, Logical Constants across Varying Types, *Notre Dame Journal of Formal Logic*, vol. 30, no. 3, p. 315-342, 1989.
- [2] J. van Benthem & D. Bonnay, Modal logic and Invariance, *Journal of Applied and Non-classical Logics*, 18 (2-3), pp. 153-173, 2008.
- [3] D. Bonnay, Logicality and Invariance, *Bulletin of Symbolic Logic*, 2008, 14, 1: 29-68
- [4] M. Dummett *The Logical Basis of Metaphysics*, Harvard University Press, 1991.
- [5] S. Feferman, Logic, logics, and logicism, *Notre Dame Journal of Formal Logic*, vol. 40, no. 1, 1999, pp. 31–54.
- [6] V. McGee, Logical operations, *Journal of Philosophical Logic*, vol. 25, 1996, pp. 567–580.
- [7] G. Sambin, G. Battilotti & C. Faggian Basic Logic : reflection, symmetry, visibility, *Journal of Symbolic Logic*, vol. 65, p. 979-1013, 2000.
- [8] G. Sher, *The bounds of logic*, MIT Press, 1991.
- [9] A. Tarski, What are logical notions?, *History and Philosophy of Logic*, v. 7, 1986, pp. 143–154.
- [10] D. Westerståhl (1989) Quantifiers in formal and natural languages, 1989, in D. Gabbay & F. Guentner (eds), *Handbook of Philosophical Logic*, vol. IV, Reidel, Dordrecht.

Items [2] and [3] are included. Course materials will be made available on-line during the week of the course. They will be accessible from here: <http://lumiere.ens.fr/~dbonnay/>